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SOIL-BEAM INTERACTION ANALYSIS WITH A TWO-PARAMETER LAYER SOIL MODEL: HOMOGENEOUS MEDIUM

by

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Development of an economical and analytical tool for determining and analyzing the response of a beam resting on a ground surface has been the aim of this study. This was accomplished by a simple mechanical soil model used to formulate the responses of the beam and the ground surface. The two parameters of this model can be defined from the information of the soil profile alone for any of the more complex profiles. From this mechanical soil

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20, ABSTRACT (Continued).

model, analytical expressions were obtained for responses of the beam lying over a ground surface and subjected to vertical loads. These expressions were then coded into the computer program, 2PARM. The results were compared with computed results obtained by a finite element method.

The study has maintained that this model and computer program can well predict the responses of the beam bearing on the surface of an elastic homogeneous medium.

PREFACE

This report presents investigations and results of utilizing a two-parameter layer soil model for solving soil-structure interaction problems. The two-parameter models are capable of coupling the responses of the soil to the structure, yet are simple enough to be developed as analytical expressions in an easy-to-use computer program.

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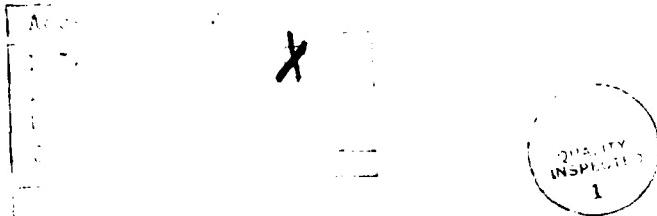
CONTENTS

CONVERSION FACTORS, NON-SI TO SI (METRIC)
UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
foot-pounds (force)	1.355818	metre-newtons or joules
kips (force)	4.448222	kilonewtons
kip (force)-feet	1355.818	newton-metres
kips (force) per square foot	47.88026	kilopascals
kips (force) per square inch	6.894757	megapascals
pounds (force)	4.448222	newtons
pounds (force) per square foot	47.88026	pascals
pounds (force) per square inch	6.894757	kilopascals

SOIL-BEAM INTERACTION ANALYSIS WITH A TWO-PARAMETER
LAYER SOIL MEDIUM: HOMOGENEOUS MEDIUM

PART I: INTRODUCTION

One-Parameter Versus Two-Parameter Models

1. In the analysis of a beam bearing on a ground surface, the soil medium is often idealized by a Winkler one-parameter model, significantly simplifying the representation of the ground behavior. This idealization makes it possible to develop analytical expressions for the beam-soil system for a variety of loading and boundary conditions. Unfortunately, the soil response at one point is uncoupled from the displacement at other points in the Winkler model, and the model fails to reproduce the characteristics of a continuous medium. To resolve this problem, two-parameter models have been developed (Filonenko-Borodich 1940, Pasternak 1954). These models are capable of coupling the response within the soil, yet simple enough to permit development of analytical expressions for the beam-soil system. This is accomplished with more or less the same degree of difficulty as that associated with the Winkler model.

Parameter Determination

2. The parameters of the model can be determined from known displacements of the ground surface (Kerr 1976) or by using a variational method together with an assumed displacement profile of the ground with depth (Vlasov and Leonteve 1966, Harr et al. 1969). However, since the ground surface displacement and the displacement profile are not known before the analysis, it is difficult to determine the parameters for the analysis. When the parameters are defined from the ground displacements for a particular case, they may not be applicable to other cases. Nonhomogeneity of the soil medium makes the above stated difficulty even more pronounced. Thus, in the present form of the two-parameter models, it is not only inconvenient to determine the model parameters, but also is very difficult when the soil profile is the only information available.

Simple Mechanical Soil Model

3. The objective of the study is to develop an economical, analytical tool for the response analysis of a beam resting on ground surface. To achieve this objective, a simple mechanical soil model is developed and used in formulating the responses of the beam resting on a ground surface. The developed model is classified as a two-parameter model. Contrary to conventional two-parameter models, however, the parameters of the new model can be defined from the information of the soil profile alone for any complex profile.

PART II: TWO-PARAMETER MODEL FOR LAYERED MEDIUM

4. A horizontal soil deposit, underlain by a rigid base, is idealized
as a system made of elastic vertical columns as shown in Figure 1. The system
is assumed to be subjected to vertical loads at the surface and to be under
plane-strain conditions (e.g., the columns and loads are infinitely long in
the direction perpendicular to the cross section shown in Figure 1). The
forces induced in the system are axial forces in the column and friction
forces along the vertical sides of the column. For an infinitely small width
of the beam element, dx , the axial normal stress and surface shear stress
are expressed by, respectively,

$$\sigma(x,z) = E' \frac{\partial w(x,z)}{\partial z}$$

$$\tag{1}$$

$$\tau(x,z) = G' \frac{\partial w(x,z)}{\partial z}$$

where

σ and τ = shear stress along the sides of the column and axial
normal stress in the column

w = vertical displacement of the column

E' and G' = constants of the column associated with axial force and
friction force

dx = cross-section area of the column

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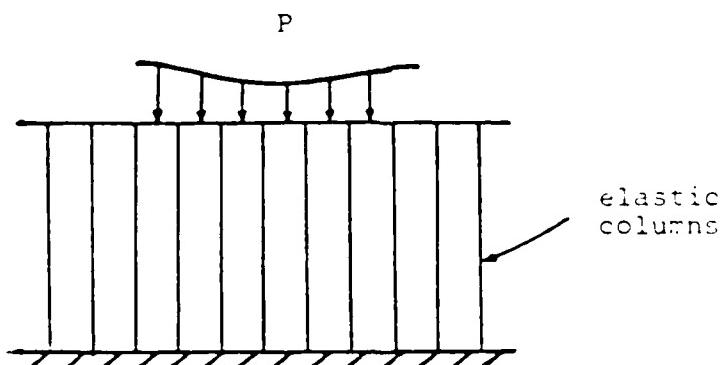
elastic
columns

Figure 1. Soil medium idealized
by a system of elastic columns

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Two Approaches

5. The displacement $w(x,z)$ in the system is generally expressed by

$$w(x,z) = \sum_n \phi_n(z) W_n(x) \tag{2}$$

where

$\phi_n(z)$ = assumed function of z

$W_n(x)$ = unknown vertical displacement

In order to define $\phi_n(z)$, two approaches were considered in the study. In the first approach, orthogonal functions are selected for $\phi_n(z)$ which can satisfy the homogeneous boundary conditions at both ends of the column. The displacement is the resultant of superpositions of various shapes, $\phi_n(z)$, weighted by $W_n(x)$ in this approach.

Second Approach Preferred

6. In the second approach, a piece-wise linear displacement is assumed along the column. Thus, the column is divided into a number of segments and one linear function is assumed within a segment. The second approach is more convenient and computationally more efficient than the first approach. The expressions and computer program presented in this report are based on the second approach, while the expressions based on the first approach are given in a separate volume.

7. Inside the model, a horizontal layer with thickness, ℓ , is considered as shown in Figure 2. The column element in this layer is subjected to vertical forces as also shown in Figure 2. The work done by the internal and external forces for this element are expressed by, respectively,

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$$W_i = - \int_0^{\ell} \sigma \, \frac{\partial w(x,z)}{\partial z} \, dx$$

$$W_e = \{p_u(x) \, w(x,0) + p_{\ell}(x) \, w(x,\ell)\} \, dx - \int_0^{\ell} \tau \, w(x,z) \, dz \tag{3}$$

$$+ \int_0^{\ell} \tau + \frac{\partial \tau}{\partial x} \, dx \quad w(x,z) \, dz$$

where

W_i and W_e = work done by internal and external forces

p_u and p_{ℓ} = vertical forces acting on a unit area of the upper and lower surfaces of the layer

σ and τ = stresses explained in equation 1

Substituting equation 3 into the equilibrium conditions expressed by $W_i + W_e = 0$, the following equation is obtained:

$$- \int_0^{\ell} \frac{\partial \tau}{\partial x} \, w(x,z) \, dz \, dx + \int_0^{\ell} \sigma \, \frac{\partial w(x,z)}{\partial x} \, w(x,z) \, dz \, dx$$

$$\tag{4}$$

$$= p_u(x) \, w(x,0) \, dx + p_{\ell}(x) \, w(x,\ell) \, dx$$
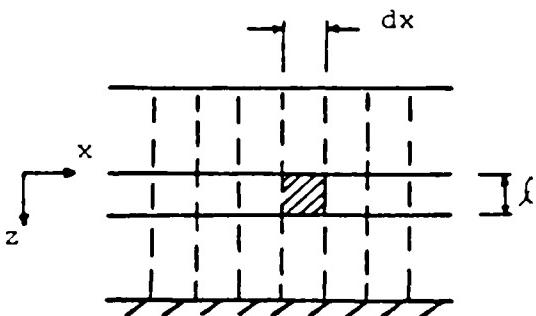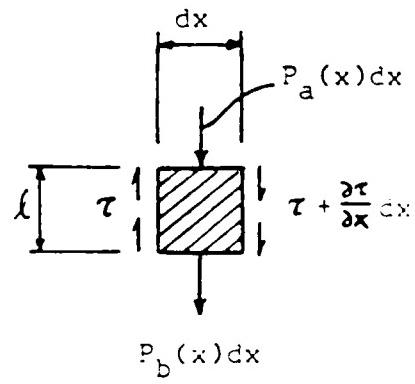
a. Horizontal layer   b. Column element subjected to forces

Figure 2. Horizontal layer and column element in the layer subjected to forces

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8. When a continuous linear displacement is assumed within the layer considered, $w(x,z)$ in equation 2 can be expressed by

$$w(x,z) = \begin{Bmatrix} \phi_u(z) \\ \phi_\ell(z) \end{Bmatrix}^T \begin{Bmatrix} W_u(x) \\ W_\ell(x) \end{Bmatrix} \tag{5}$$

where W_u and W_ℓ = vertical displacements of the layer at upper and lower ends, respectively; and

$$\begin{Bmatrix} \phi_u(z) \\ \phi_\ell(z) \end{Bmatrix} = \begin{Bmatrix} 1 - \dfrac{z}{\ell} \\ \dfrac{z}{\ell} \end{Bmatrix} \tag{6}$$

Substituting equation 1 into equation 4 and using the relationship given in equation 5, equation 4 is rewritten by

$$- [N^L] \begin{Bmatrix} W_u''(x) \\ W_\ell''(x) \end{Bmatrix} + [K^L] \begin{Bmatrix} W_u(x) \\ W_\ell(x) \end{Bmatrix} = \begin{Bmatrix} p_u(x) \\ p_\ell(x) \end{Bmatrix} \tag{7}$$

where $W'' = d^2W/dx^2$; $[N^L]$ and $[K^L]$ are defined by the material constants of the column and are, respectively,

$$[N^L] = \int_0^\ell G'(z) \begin{Bmatrix} \phi_u(z) \\ \phi_\ell(z) \end{Bmatrix} \begin{Bmatrix} \phi_u(z) \\ \phi_\ell(z) \end{Bmatrix}^T dz$$

$$\tag{8}$$

$$[K^L] = \int_0^\ell E'(z) \begin{Bmatrix} d\phi_u(z)/dz \\ d\phi_\ell(z)/dz \end{Bmatrix} \begin{Bmatrix} d\phi_u(z)/dz \\ d\phi_\ell(z)/dz \end{Bmatrix}^T dz$$

Assuming linear variation of G' and E' with depth z within a layer, the above integrals are evaluated as

$$[N^L] = \frac{G'(0)\ell}{12} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} + \frac{G'(\ell)\ell}{12} \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$[K^L] = \frac{E'(0)}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{E'(\ell)}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{9}$$

where

$G'(0)$ and $E'(0)$ = material constants of the columns at the upper surface of the layer considered

$G'(\ell)$ and $E'(\ell)$ = material constants of the columns at the lower surface of the layer considered

9. When the system is divided into N horizontal layers, the equilibrium and compatibility conditions at the interfaces between the layers lead to

$$-[N]\{W''(x)\} + [K]\{W(x)\} = \{P(x)\} \tag{10}$$

where

$[N]$ and $[K]$ = $N \times N$ matrices constructed from $[N^L]$ and $[K^L]$ as shown in Figure 3

$[W]$ = vector containing vertical displacements at N interfaces, in which the first location is assigned for the displacement at the ground surface

$[P]$ = vectors containing external forces applied at the N interfaces, in which the first location is assigned for the force applied at the ground surface. The expression is identical to the governing equation of commonly used two-parameter models except in a matrix form

10. When a beam is lying horizontally over the ground surface and is subject to vertical forces, the behavior of the horizontal beam is governed by the equation expressed by

$$EI \, W_b^{IV}(x) = P(x) - P_s(x) \tag{11}$$

where

EI = flexural rigidity of the beam

W_b = vertical displacement of horizontal beam

$W_b^{IV} = d^4 W_b / dx^4$

P_s = soil reaction force at the soil-beam interface

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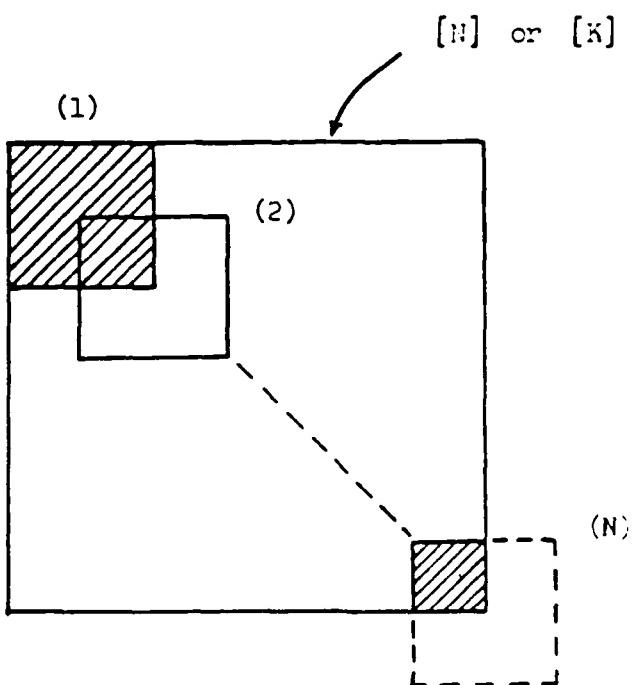
Figure 3. Construction of [N] and [K]
from [N^L] and [K^L]

Using equations 10 and 11, equilibrium and compatibility conditions at the soil-beam interface lead to

$$[EI] \{W^{IV}(x)\} - [N] \{W''(x)\} + [K] \{W(x)\} = \{P(x)\} \tag{12}$$

where [EI] = diagonal matrix containing EI at the location (1, 1) and zeros at other locations; and {P} = force vector containing the applied vertical load, at the first location in the vector and zeros at other locations.

PART III: BOUNDARY-VALUE PROBLEMS

11. Idealizing the soil medium as a mechanical model as presented in Part II, the analytical expressions for the responses of the ground and the responses of the beam bearing on a ground surface are obtained solving equations 10 and 12, respectively. The loading conditions considered are vertical loads applied on ground surface and vertical and moment forces applied to the beam.

Responses of Ground Without Beam

12. The expression of the response for any distributed load can be obtained by integrating the expression for a line load. The solution for a line load can be obtained by solving equation 10 with $\{P\} = \{0\}$ and treating a line load as one of the boundary conditions.

General solution
for homogeneous equations

13. The solution for equation 10 with $\{P\} = \{0\}$ is expressed by

$$\{W(x)\} = \sum_{n=1}^{N} C_{1n} \, e^{\lambda_n x} \{d_n\} + C_{2n} \, e^{-\lambda_n x} \{d_n\}$$

$$= [A(+x)]\{C_1\} + [A(-x)]\{C_2\}$$

(13)

where

$\{C_1\}$ and $\{C_2\}$ = vectors containing the unknown constants

$[A(+x)]$ and $[A(-x)]$ = $N \times N$ matrices in which the nth columns are $e^{\lambda_n x}\{d_n\}$ and $e^{\lambda_n x}\{d_n\}$

λ_n and $\{d_n\}$ = n^{th} eigenvalue and eigenvectors obtained from the following characteristic equation obtained from equation 10

$$[-\lambda^2[N] + [K]] \{d\} = \{0\}$$

(14)

With equation 13, the vertical shear force along the vertical plane are obtained as

$$\{T_s(x)\} = [N]\{W'(x)\}$$

$$= [N]\{[A'(+x)]\{C_1\} + [A'(-x)]\{C_2\}\} \tag{15}$$

where

$$[A'(\pm x)] = \left[\frac{d}{dx} A(\pm x)\right]; \text{ and } \{T_s(x)\} = \text{vertical shear force}$$

Response due to a line load
applied on a ground surface

14. The soil medium is divided into left-hand and right-hand sides with respect to the location of a line load applied as shown in Figure 4. Since
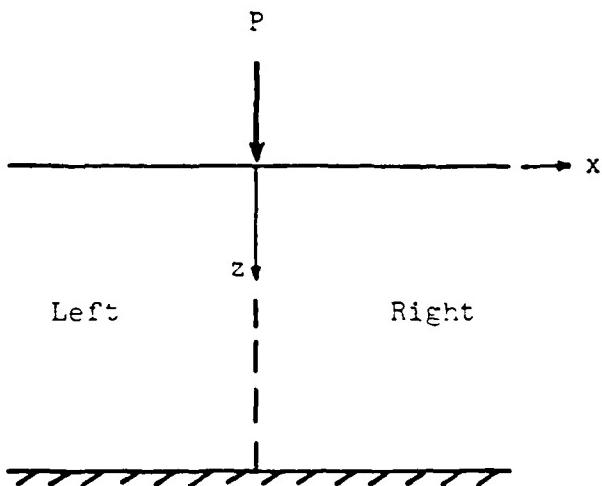
Figure 4. Line load applied on the surface

the soil responses should diminish with distance, the displacements and forces induced in the soil medium (Equations 13 and 15) are deduced to the following expressions:

Left-hand side

$$\{W(x)\}_L = [A(+x)]\{C\}_L \quad \text{and} \quad \{T_s(x)\}_L = [N][A'(+x)]\{C\}_L$$

Right-hand side   (16)

$$\{W(x)\}_R = [A(-x)]\{C\}_R \quad \text{and} \quad \{T_s(x)\}_R = [N][A'y(-x)]\{C\}_R$$

where the subscripts L and R indicate the left-hand and right-hand sides, respectively. The boundary conditions at $x = 0$ require

$$\{W(0)\}_L = \{W(0)\}_R$$

(17)

$$\{T_s(0)\}_L - \{T_s(0)\}_R = \{P\}$$

15. Substituting equation 16 into equation 17 to determine the unknown constants, the displacements of the soil medium are expressed by

$$\{W(x)\} = \frac{P}{2}\left[A(-|x|)\right]\{\gamma\}$$

(18)

where $\{\gamma\}$ = vector containing the numbers located at the first column of the matrix, $[[N][A'(-0)]]^{-1}$. Thus, the displacement of the ground surface subjected to a line load at $x = \xi$ is expressed from equation 18 as

$$W_1(x) = \frac{P(\xi)}{2} \sum_{n=1}^{N} e^{-\lambda_n |x-\xi|} \gamma_n$$

(19)

where

W_1 = displacement of the ground surface (e.g., the value located at the first location $\{W\}$

γ_n = the value located at the nth location in $\{\gamma\}$

14

Response due to a trapezoidal
load applied on ground surface

16. Using the expression in equation 19, the surface displacement due to a distributed load applied at the surface is obtained as

$$W_1(x) = \int_{x_1}^{x_2} \frac{P(\xi)}{2} \sum_{n=1}^{N} e^{-\lambda_n |x-\xi|} \gamma_n d\xi \tag{20}$$

If the distributed load is a trapezoidal load expressed by $P(\xi) = a\xi + b$, equation 20 yields the following expression:

$$W_1(x) = \sum_{n=1}^{N} A_n e^{-\lambda_n |x-x_1|} + B_n e^{-\lambda_n |x-x_2|} + C_n + D_n \tag{21}$$

where

$$A_n = \frac{\gamma_n}{\lambda_n} \left\{ a\left(x_1 + \lambda_n^{-1}\right) + b \right\}, \quad B_n = -\frac{\gamma_n}{\lambda_n} \left\{ a\left(x_1 + \lambda_n^{-1}\right) + b \right\}$$

$$C_n = 0 \quad \text{and} \quad D_n = 0 \qquad\qquad \text{for} \quad x \le x_1$$

$$A_n = \frac{-\gamma_n}{\lambda_n} \left\{ a\left(x_1 - \lambda_n^{-1}\right) + b \right\}, \quad B_n = \frac{-\gamma_n}{\lambda_n} \left\{ a\left(x_2 - \lambda_n^{-1}\right) + b \right\}$$

$$\tag{22}$$

$$C_n = 2a\lambda_n^{-1}\gamma_n \quad \text{and} \quad D_n = 2b\lambda_n^{-1}\gamma_n \qquad\qquad \text{for} \quad x_1 \le x \le x_2$$

$$A_n = \frac{-\gamma_n}{\lambda_n} \left\{ a\left(x_1 - \lambda_n^{-1}\right) + b \right\}, \quad B_n = \frac{-\gamma_n}{\lambda_n} \left\{ a\left(x_2 - \lambda_n^{-1}\right) + b \right\}$$

$$C_n = 0 \quad \text{and} \quad D_n = 0 \qquad\qquad \text{for} \quad x \ge x_2$$

Responses of Beam Bearing on Ground Surface

General solution of
homogeneous equations

17. In order to obtain the responses of the beam bearing on the ground surface, equation 11 with $\{P\} = \{0\}$ must be solved. The characteristic equation obtained from this homogeneous equation is

$$[\lambda^4 [EI] - \lambda^2 [n] + [K]] \{d\} = \{0\} \tag{23}$$

N sets of four complex eigenvalues and two complex eigenvectors are found from equation 23 in general. The nth four eigenvalues are $+\lambda_n$, $-\lambda_n$, $+\lambda'_n$ and $-\lambda'_n$ in which λ_n and λ'_n are conjugate pair. The eigenvectors corresponding to $\pm\lambda_n$ and $\pm\lambda'_n$ are also conjugate pair and denoted as $\{d_n\}$ and $\{d'_n\}$, respectively. Thus, the solution of the homogeneous equation above considered is expressed by

$$\{W(x)\} = \sum_{n=1}^{N} C_{1n} e^{\lambda_n x} \{d_n\} + C_{2n} e^{-\lambda_n x} \{d_n\} + C_{3n} e^{\lambda'_n x} \{d'_n\}$$

$$+ C_{4n} e^{-\lambda'_n x} \{d'_n\} \tag{24}$$

where C_{1n}, C_{2n}, C_{3n}, and C_{4n} = unknown constants. Combing between the first and fourth terms and between the second and third terms in equation 24, this equation can be rewritten as

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$$\{W(x)\} = \sum_{n=1}^{N} C_{1n} \, e^{\alpha_n x} \left\{ \cos(\beta_n x) \left\{ d_n^r \right\} - \sin(\beta_n x) \left\{ d_n^i \right\} \right\}$$

$$+ C_{2n} \, e^{\alpha_n x} \left\{ \cos(\beta_n x) \left\{ d_n^i \right\} + \sin(\beta_n x) \left\{ d_n^r \right\} \right\}$$

$$+ C_{3n} \, e^{-\alpha_n x} \left\{ \cos(\beta_n x) \left\{ d_n^r \right\} + \sin(\beta_n x) \left\{ d_n^i \right\} \right\} \tag{25}$$

$$+ C_{4n} \, e^{-\alpha_n x} \left\{ \cos(\beta_n x) \left\{ d_n^i \right\} - \sin(\beta_n x) \left\{ d_n^r \right\} \right\}$$

$$= [A(+x)] \{C_1\} + [B(+x)] \{C_2\} + [A(-x)] \{C_3\} + [B(-x)] \{C_4\}$$

where $\{d_n\} = \left\{ d_n^r \right\} + i \left\{ d_n^i \right\}$; the nth columns in $[A(\pm x)]$ and $[B(\pm x)] =$ $e^{\pm \alpha_n x} \left\{ \cos(\beta_n x) \left\{ d_n^r \right\} \pm \sin(\beta_n x) \left\{ d_n^i \right\} \right\}$ and $e^{\pm \alpha_n x} \cos(\beta_n x) \left\{ d_n^i \right\}$ $\pm \sin(\beta_n x) \left\{ d_n^r \right\}$, respectively; and $\lambda_n = \alpha_n + i\beta_n$.

18. When a horizontal beam exists only at the ground surface, just one set of four eigenvalues and two eigenvectors have an imaginary part and all other sets (N-1 sets) do not have such a part. Thus, equation 25 is deduced to

$$\{W(x)\} = [A(+x)] \{C_1\} + [A(-x)] \{C_3\} + C_2 \{B(+x)\} + C_4 \{B(-x)\} \tag{26}$$

where, setting the complex eigenvalue as the first eigenvalue (e.g., $\lambda_1 = \alpha_1 + i\beta_1$, $\lambda_2 = \alpha_2$, $\lambda_3 = d_3$, ---), $\{B(\pm x)\}$ is expressed by

$$\{B(\pm x)\} = e^{\pm \alpha_1 x} \left\{ \cos(\beta_1 x) \left\{ d_1^i \right\} \pm \sin(\beta_1 x) \left\{ d_1^r \right\} \right\} \tag{27}$$

Thus, the vertical displacements and shear forces induced in the soil model can be expressed in the following forms, respectively,

17

$$\{W(x)\} = [AB(+x)] \begin{Bmatrix} \tilde{C}_1 \\ C_2 \end{Bmatrix} + [AB(-x)] \begin{Bmatrix} \tilde{C}_3 \\ C_4 \end{Bmatrix}$$

(28)

$$\{T_s(x)\} = [N][AB'(+x)] \begin{Bmatrix} \tilde{C}_1 \\ C_2 \end{Bmatrix} + [N][AB'(-x)] \begin{Bmatrix} \tilde{C}_3 \\ C_4 \end{Bmatrix}$$

where

$$[AB(\pm x)] = [[A(\pm x)] \mid \{B(\pm x)\}]$$

$$[AB'(\pm x)] = \left[\frac{d}{dx} \ AB(\pm x) \right]$$

(29)

$$\tilde{C}_1 = \{C_1\} \quad \text{and} \quad \tilde{C}_3 = \{C_3\}$$

Since the vertical displacement of the beam, $W_b(x)$, is located at the first value in $\{W(x)\}$, the displacement and forces induced in the beam are expressed by

$$W_b(x) = \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB(+x)] \begin{Bmatrix} \tilde{C}_1 \\ C_2 \end{Bmatrix} + \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB(-x)] \begin{Bmatrix} \tilde{C}_3 \\ C_4 \end{Bmatrix}$$

$$\theta_b(x) = \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'(+x)] \begin{Bmatrix} \tilde{C}_1 \\ C_2 \end{Bmatrix} + \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'(-x)] \begin{Bmatrix} \tilde{C}_3 \\ C_4 \end{Bmatrix}$$

(30)

$$M_b(x) = -EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB''(+x)] \begin{Bmatrix} \tilde{C}_1 \\ C_2 \end{Bmatrix} - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB''(-x)] \begin{Bmatrix} \tilde{C}_3 \\ C_4 \end{Bmatrix}$$

$$T_b(x) = -EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'''(+x)] \begin{Bmatrix} \tilde{C}_1 \\ C_2 \end{Bmatrix} - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'''(-x)] \begin{Bmatrix} \tilde{C}_3 \\ C_4 \end{Bmatrix}$$

where

W_b and θ_b = vertical and rotational displacements of the beam

M_b and T_b = moment and shear forces induced in the beam

$$\tilde{0} = \text{vector with the size } N - 1 \text{ containing all zeros}$$

$$[AB'(\pm x)] = \left[\frac{d}{dx} AB(\pm)\right]$$

$$[AB''(\pm)] = \left[\frac{d^2}{dx^2} AB(\pm)\right]$$

$$[AB'''(\pm)] = \left[\frac{d^3}{dx^3} AB(\pm x)\right]$$

Equations 28 and 30 are combined in the following manner:

$$\begin{Bmatrix} \tilde{W}(x) \\ \theta_b(x) \end{Bmatrix} = [D(+x)] \begin{Bmatrix} \tilde{C}_1 \\ C_2 \end{Bmatrix} + [D(-x)] \begin{Bmatrix} \tilde{C}_3 \\ C_4 \end{Bmatrix}$$

$$\begin{Bmatrix} \tilde{T}(x) \\ M_b(x) \end{Bmatrix} = [E(x)] \begin{Bmatrix} \tilde{C}_1 \\ C_2 \end{Bmatrix} + [E(-x)] \begin{Bmatrix} \tilde{C}_3 \\ C_4 \end{Bmatrix} \tag{31}$$

where

$$\tilde{W}(x) = \{W(x)\}$$

$$\tilde{T}(x) = \{T_s(x)\} + \begin{Bmatrix} T_b(x) \\ \tilde{0} \end{Bmatrix}$$

$$[D(\pm x)] = \begin{bmatrix} \dfrac{[AB(\pm x)]}{\begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'(\pm x)]} \end{bmatrix} \tag{32}$$

$$[E(\pm x)] = \begin{bmatrix} [N][AB'(\pm x)] - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'''(\pm x)] \\ \hline -EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB''(\pm)] \end{bmatrix}$$

19

Responses due to a line load
applied on an infinitely long beam

19. Vertical and moment loads are applied at the location $x = 0$ of an infinitely long beam lying over the ground surface. Since the responses must diminish with distance, the expressions given in equations 28, 30, and 31 contain only the first and second terms for $x \leq 0$ and $x \geq 0$, respectively. The boundary conditions at $x = 0$ requires

$$\begin{Bmatrix} \tilde{w}(0) \\ \theta_b(0) \end{Bmatrix}_L = \begin{Bmatrix} \tilde{w}(0) \\ \theta_b(0) \end{Bmatrix}_R \tag{33}$$

$$\begin{Bmatrix} \tilde{T}(0) \\ M_b(0) \end{Bmatrix}_L - \begin{Bmatrix} \tilde{T}(0) \\ M_b(0) \end{Bmatrix}_R = \begin{Bmatrix} \tilde{P} \\ M \end{Bmatrix}$$

where

        Subscript L and R = left-hand and right-hand sides with respect to the location of the load applied

        $\tilde{P}$ = load vector containing the applied load at the first location of the vector and zeros at other locations of the vector

        M = applied moment load

Substituting equation 31 into equation 33, the unknown constants are found as:

$$\begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix}_L = \begin{Bmatrix} C_3 \\ C_4 \end{Bmatrix}_R = [\gamma] \begin{Bmatrix} \tilde{P} \\ M \end{Bmatrix} \tag{34}$$

where $[\gamma] = (N+1) \times (N+1)$ matrix expressed by

$$[\gamma] = [\ [E(+0)]\ [D(+0)]^{-1}\ [D(-0)] - [E(-0)]\ ]^{-1}$$

$$\tag{35}$$

$$= [\ [E(+0)] - [E(-0)]\ [D(-0)]^{-1}\ [D(-0)]\ ]^{-1}$$

Thus, substituting the constants expressed in equation 34 into equation 28, the responses of the soil medium for a line load applied at x are expressed by

20

$$\{W(x)\} = P(\xi)\left[AB(-|x - \xi|)\right]\{\gamma_1\} + M(\xi)\left[AB(-|x - \xi|)\right]\{\gamma_{N+1}\}$$

(36)

$$\{T_s(x)\} = P(\xi)[N]\left[AB'(-|x - \xi|)\right]\{\gamma_1\} + M(\xi)[N]\left[AB'(-|x - \xi|)\right]\{\gamma_{N+1}\}$$

where $\{\gamma_1\}$ and $\{\gamma_{N+1}\}$ = vector located at the first and $N+1$ the columns in the matrix $[\gamma]$, respectively. Similarly, substituting them into equation 30, the responses of the beam for a line load applied at $x = \xi$ are expressed by

$$W_b(x) = P(\xi)\begin{Bmatrix}1\\ \tilde{0}\end{Bmatrix}^T\left[AB(-|x - \xi|)\right]\{\gamma_1\} + M(\xi)\begin{Bmatrix}1\\ \tilde{0}\end{Bmatrix}^T\left[AB(-|x - \xi|)\right]\{\gamma_{N+1}\}$$

$$\theta_b(x) = P(\xi)\begin{Bmatrix}1\\ \tilde{0}\end{Bmatrix}^T\left[AB'(-|x - \xi|)\right]\{\gamma_1\} + M(\xi)\begin{Bmatrix}1\\ \tilde{0}\end{Bmatrix}^T\left[AB'(-|x - \xi|)\right]\{\gamma_{N+1}\}$$

(37)

$$M_b(x) = -EI[P(\xi)\begin{Bmatrix}1\\ \tilde{0}\end{Bmatrix}^T\left[AB''(-|x - \xi|)\right]\{\gamma_1\} + M(\xi)\begin{Bmatrix}1\\ \tilde{0}\end{Bmatrix}^T\left[AB'(-|x - \xi|)\right]\{\gamma_{N+1}\}$$

$$T_b(x) = -EI[P(\xi)\begin{Bmatrix}1\\ \tilde{0}\end{Bmatrix}^T\left[AB'''(-|x - \xi|)\right]\{\gamma_1\} + M(\xi)\begin{Bmatrix}1\\ \tilde{0}\end{Bmatrix}^T\left[AB'''(-|x - \xi|)\right]\{\gamma_{N+1}\}$$

The expressions for $[AB]$, $[AB'']$, and $[AB''']$ are given in Appendix A.

Responses due to a trapezoidal
load applied on infinitely long beam

20. A vertical trapezoidal load, $P(\xi) = a\xi + b$, is applied over the area $x_1 \sim x_2$ of an infinitely long beam lying over the ground surface. Integrating the responses due to a line load expressed in equations 36 and 37 with $M(\xi) = 0$, the following expressions for the responses due to this load are obtained:

21

$$\{W(x)\} = a[D_a(x)] \{\gamma_1\} + b [D_b(x)] \{\gamma_1\}$$

$$\{T_s(x)\} = a[\dot{N}] [D_a'(x)] \{\gamma_1\} + b [D_b'(x)] \{\gamma_1\}$$ Soil

(38)

$$W_b(x) = a \left\{ \begin{matrix} 1 \\ \tilde{0} \end{matrix} \right\}^T [D_a(x)] \{\gamma_1\} + b \left\{ \begin{matrix} 1 \\ \tilde{0} \end{matrix} \right\}^T [D_b(x)] \{\gamma_1\}$$

$$\theta_b(x) = a \left\{ \begin{matrix} 1 \\ \tilde{0} \end{matrix} \right\}^T [D_a'(x)] \{\gamma_1\} + b \left\{ \begin{matrix} 1 \\ \tilde{0} \end{matrix} \right\}^T [D_b'(x)] \{\gamma_1\}$$

$$M_b(x) = -EI \left[a \left\{ \begin{matrix} 1 \\ \tilde{0} \end{matrix} \right\}^T [D_a''(x)] \{\gamma_1\} + b \left\{ \begin{matrix} 1 \\ \tilde{0} \end{matrix} \right\}^T [D_b''(x)] \{\gamma_1\} \right]$$ Beam

$$T_b(x) = -EI \left[a \left\{ \begin{matrix} 1 \\ \tilde{0} \end{matrix} \right\}^T [D_a'''(x)] \{\gamma_1\} + b \left\{ \begin{matrix} 1 \\ \tilde{0} \end{matrix} \right\}^T [D_b'''(x)] \{\gamma_1\} \right]$$

where

$$[D_a] = \int_{x_1}^{x_2} \xi [AB(- |x - \xi|)] \, d\xi \ , \quad [D_b] = \int_{x_1}^{x_2} [AB(- |x - \xi|)] \, d\xi \ ,$$

$$[D_a'] = \int_{x_1}^{x_2} \xi [AB'(- |x - \xi|)] \, d\xi \ , \quad [D_b'] = \int_{x_1}^{x_2} [AB'(- |x - \xi|)] \, d\xi \ ,$$

(39)

$$[D_a''] = \int_{x_1}^{x_2} \xi [AB''(- |x - \xi|)] \, d\xi \ , \quad [D_b''] = \int_{x_1}^{x_2} [AB''(- |x - \xi|)] \, d\xi \ ,$$

$$[D_a'''] = \int_{x_1}^{x_2} \xi [AB'''(- |x - \xi|)] \, d\xi \ , \quad [D_b'''] = \int_{x_1}^{x_2} [AB'''(- |x - \xi|)] \, d\xi \ ,$$

The expressions for $[D_a] \sim [D_a''']$ and $[D_b] \sim [D_b''']$ in equation 38 are given in Appendix A.

Responses due to loads
applied on a finite beam

21. A finite beam resting on a horizontal ground surface is now considered. The soil medium is divided into left, central, and right areas as shown in Figure 5. In the central area, the responses are obtained by superimposing

Figure 5. Beam resting on
soil medium
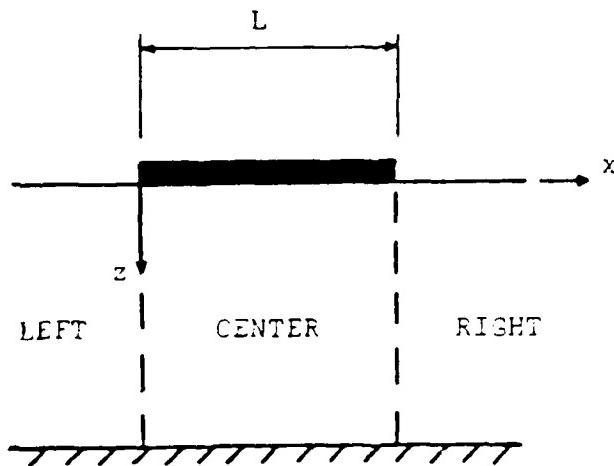
the responses for an infinitely long beam and the responses due to the effects of a finite length of the beam. Thus, the responses of a system made of a finite beam and soil are written by

$$\{W(x)\}_C = \{W^e(x)\} + \{W^i(x)\} \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\} \text{Soil}$$

$$\{T_s(x)\}_C = \{T_s^e(x)\} + \{T_s^i(x)\}$$

$$W_b(x) = W_b^e(x) + W_b^i(x) \tag{40}$$

$$\theta_b(x) = \theta_b^e(x) + \theta_b^i(x)$$

$$M_b(x) = M_b^e(x) + M_b^i(x) \left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} \text{Beam}$$

$$T_b(x) = T_b^e(x) + T_b^i(x)$$

23

where the superscripts i and e indicate, respectively, the responses for an infinitely long beam and the responses due to the effects of a finite length of the beam; and the subscript C indicates the responses in the central area. The expressions for the responses of an infinitely long beam are given in equations 37 and 38 for a line load applied and a trapezoidal load applied, respectively. The expressions of the responses due to a finite length are given in equations 28 and 30 in which the unknown constants are determined for the boundary conditions at both ends of the finite beam. The expressions of the soil responses in left and right areas are given in equation 16.

22. The boundary conditions at both ends of the central area are

$$\{W(0)\}_L = \{W(0)\}_C$$

$$\{W(L)\}_R = \{W(L)\}_C$$

$$\{T_s(0)\}_L - \{T_s(0)\}_C - T_b(0) \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} = \{0\} \tag{41}$$

$$\{T_s(L)\}_R - \{T_s(L)\}_C - T_b(L) \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} = \{0\}$$

$$M_b(0) = 0$$

$$M_b(L) = 0$$

The following relationships between $\{W\}$ and $\{T_s\}$ also exist:

$$\{T_s(0)\}_L = [k] \{W(0)\}_L$$

$$\{T_s(L)\}_R = -[k] \{W(L)\}_R \tag{42}$$

where

$$[k] = [A'(+0)] [A(+0)]^{-1} \tag{43}$$

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Substituting equation 42 into equation 41, after substituting the first two
boundary conditions in equation 41 into equation 42, the boundary conditions
given in equation 41 can be deduced to

$$[k] \{W(0)\}_C - \{T_s(0)\}_C - T_b(0) \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} = \{0\}$$

$$-[k] \{W(L)\}_C - \{T_s(L)\}_C - T_b(0) \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} = \{0\}$$

$$M_b(0) = 0$$

$$M_b(L) = 0$$

(44)

Thus, substituting the expressions for the displacements and forces into
equation 44, the boundary conditions finally lead to the following
simultaneous equations:

$$
2(N+1) \begin{bmatrix} [M_{11}] & [M_{12}] \\ [M_{21}] & [M_{22}] \\ \{M_3\}^T & \{M_4\}^T \\ \{M_5\}^T & \{M_6\}^T \end{bmatrix} \begin{Bmatrix} \tilde{C}_1 \\ C_2 \\ \tilde{C}_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} \tilde{P}_1 \\ \tilde{P}_2 \\ P_3 \\ P_4 \end{Bmatrix}
$$

(45)

where $\{C_1\}$, C_2 , $\{C_3\}$, and C_4 are unknown constants defined in
equation 28,

25

$$[M_{11}] = [k] \ [AB(+0)] + [N] \ [AB'(+0)] - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'''(+0)]$$

$$[M_{12}] = [k] \ [AB(-0)] + [N] \ [AB'(-0)] - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'''(-0)]$$

$$[M_{21}] = - [k] \ [AB(+L)] - [N] \ [AB'(+L)] + EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'''(+L)]$$

$$[M_{22}] = - [k] \ [AB(-L)] - [N] \ [AB'(-L)] + EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix} \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB'''(-L)]$$

$$\{M_3\}^T = - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB''(+0)]$$

$$\{M_4\}^T = - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB''(-0)] \tag{46}$$

$$\{M_5\}^T = - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB''(+L)]$$

$$\{M_6\}^T = - EI \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}^T [AB''(-L)]$$

$$\tilde{P}_1 = -[k] \ \{W^i(0)\} + \{T_s^i(0)\} + T_b^i(0) \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}$$

$$\tilde{P}_2 = [k] \ \{W^i(L)\} + \{T_s^i(L)\} + T_b^i(L) \begin{Bmatrix} 1 \\ \tilde{0} \end{Bmatrix}$$

$$P_3 = - M_b^i(0)$$

$$P_4 = - M_b^i(L)$$

26

After determining the unknown constants from equation 45 and substituting them into equations 28 and 30, the responses due to the effects of a finite length of the beam (e.g., $\{W^e\}$, $\{T^e_s\}$, W^E_b, θ^e_b, M^e_b, and T^e_b in equation 40) can be obtained. Then, these responses are added to the responses of an infinitely long beam (e.g., $\{W^i\}$, $\{T^i_s\}$, W^i_b, θ^i_b, M^i_b, and T^i_b in equation 40) to obtain the responses of a finite beam. When a number of loads are applied simultaneously, the resultants of those induced by each of the applied loads are used for the responses of an infinitely long beam which appears in equations 40 and 46.

23. The above formulations are coded in a computer program 2PARM. Information of the program is given in Appendix B.

PART IV: DETERMINATION OF THE PARAMETERS E' AND G'

Soil Medium and Axial Normal Stress

24. For the idealized soil medium as shown in Figure 1, the axial normal stress in the column and shear stress along the faces of the column are assumed to be expressed by, respectively,

$$\sigma(x,z) = F_E \frac{(1 - \nu_s)}{2(1 + \nu_s)} \frac{E_s}{(1 - 2\nu_s)} \frac{\partial w(x,z)}{\partial z}$$

and   (47)

$$\tau(x,z) = F_G \frac{E_s}{2(1 + \nu_s)} \frac{\partial w(x,z)}{\partial x}$$

where

E_s and ν_s = Young's modulus and Poisson's ratio of the soil medium

F_E and F_G = factors explained below

The above expressions without F_E and F_G correspond to those for a continuous medium under a completely restrained lateral displacement. The factors are included in the expressions so that the vertical displacements of the model are close to those of the elastic medium subjected to vertical surface load. Comparing equation 1 with equation 47, the parameters of the beam element in the model are found to be

$$E' = F_E \frac{(1 - \nu_s)}{2(1 + \nu_s)} \frac{E_s}{(1 - 2\nu_s)}$$

(48)

$$G' = F_G \frac{E_s}{2(1 - \nu_s)}$$

28

25. The shape of the surface displacement of the model is controlled by the value λ_n. Referring to equations 14 and 23, one can find that the value λ_n depends on the ratio E'/G' and thus F_E/F_G. The surface displacements of the model with $F_E/F_G = 1$ and 2 and are shown in the variations of Figure 6 for the normalized value. A close agreement between the computed values and those obtained by the finite-element method (FEM) can be seen for $F_E/F_G = 2$, in general.

Determination of F_E Values

26. Under the fixed F_E/F_G ratio, the absolute displacement of the model increases linearly with decreasing F_E without changing the displacement shape along the horizontal direction. For $F_E/F_G = 2$, the values F_E are determined so that the surface displacement of the model is identical to the displacement of the continuous medium at the center of a uniformly loaded area. These values are shown in Figure 7. It is seen in the figure that the values are rather independent of the depth of the soil medium, but dependent on a Poisson's ratio.

27. Using the factors F_E and F_G above found for a uniformly distributed load, the vertical surface displacements of the model were computed for a linearly varying distributed load. The values are shown in Figures 8a, b, and c, together with those obtained by a FEM. Good agreements of the computed displacements with those obtained by a FEM indicates that the factors F_E and F_G are rather independent of a type of the load applied on the surface.

28. From the above studies, the approximate values of F_E and F_G in equation 48 are found to be

$$F_E = 0.95 \sim 0.99 \quad \text{and} \quad F_G = 0.475 \sim 0.495 \quad \text{for} \quad \nu_s = 0 \sim 0.3$$

$$(49a)$$

less than the above values for $\nu_s = 0.3 \sim 0.45$

DISTANCE x/d
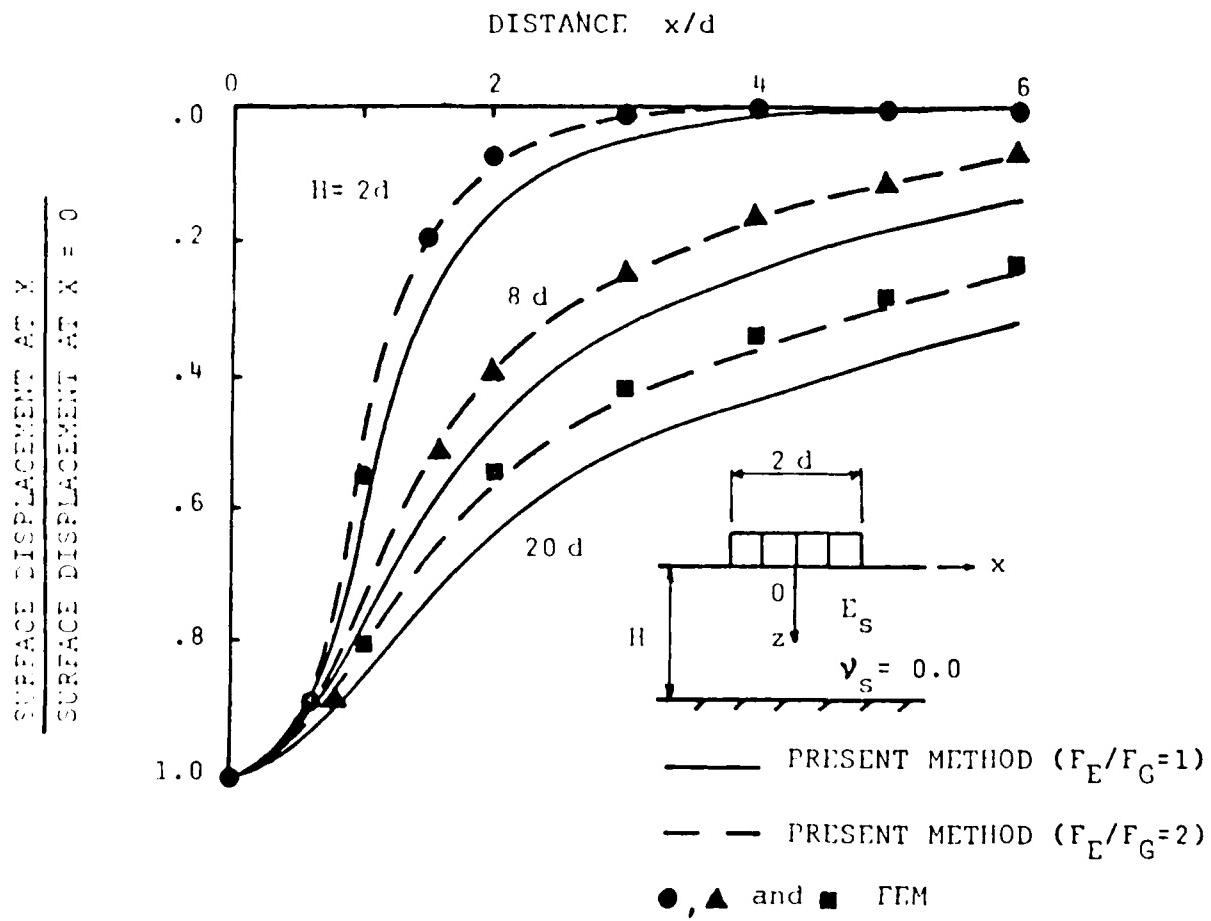
Figure 6a. Variation of ground surface displacements computed for various
values of F_E/F_G (ν_s = 0.0)

30
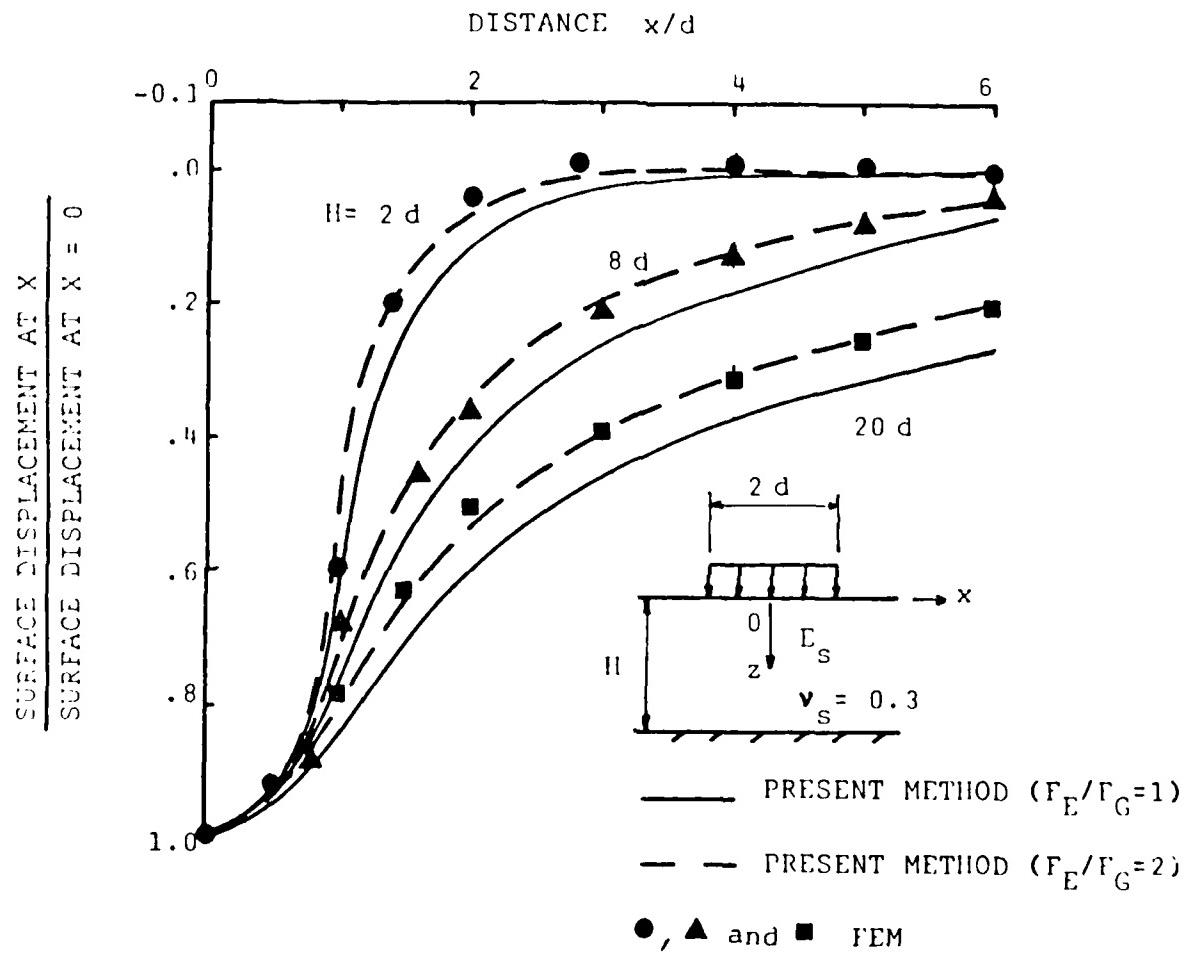
Figure 6b. Variation of ground surface displacements computed for various values of F_E/F_G ($\nu_s = 0.3$)
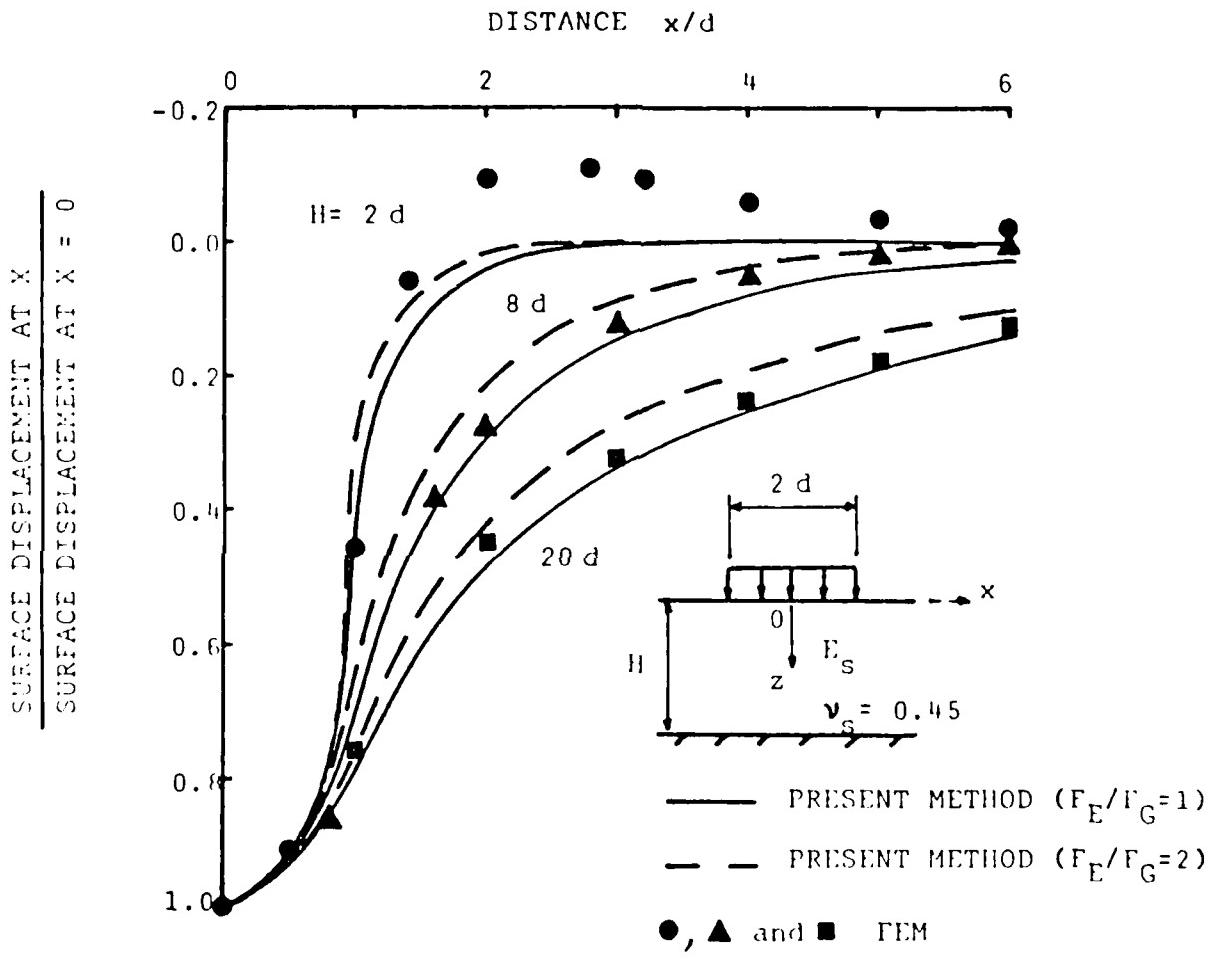
Figure 6c. Variation of ground surface displacements computed for various values of F_E/F_G ($\nu_s = 0.45$)
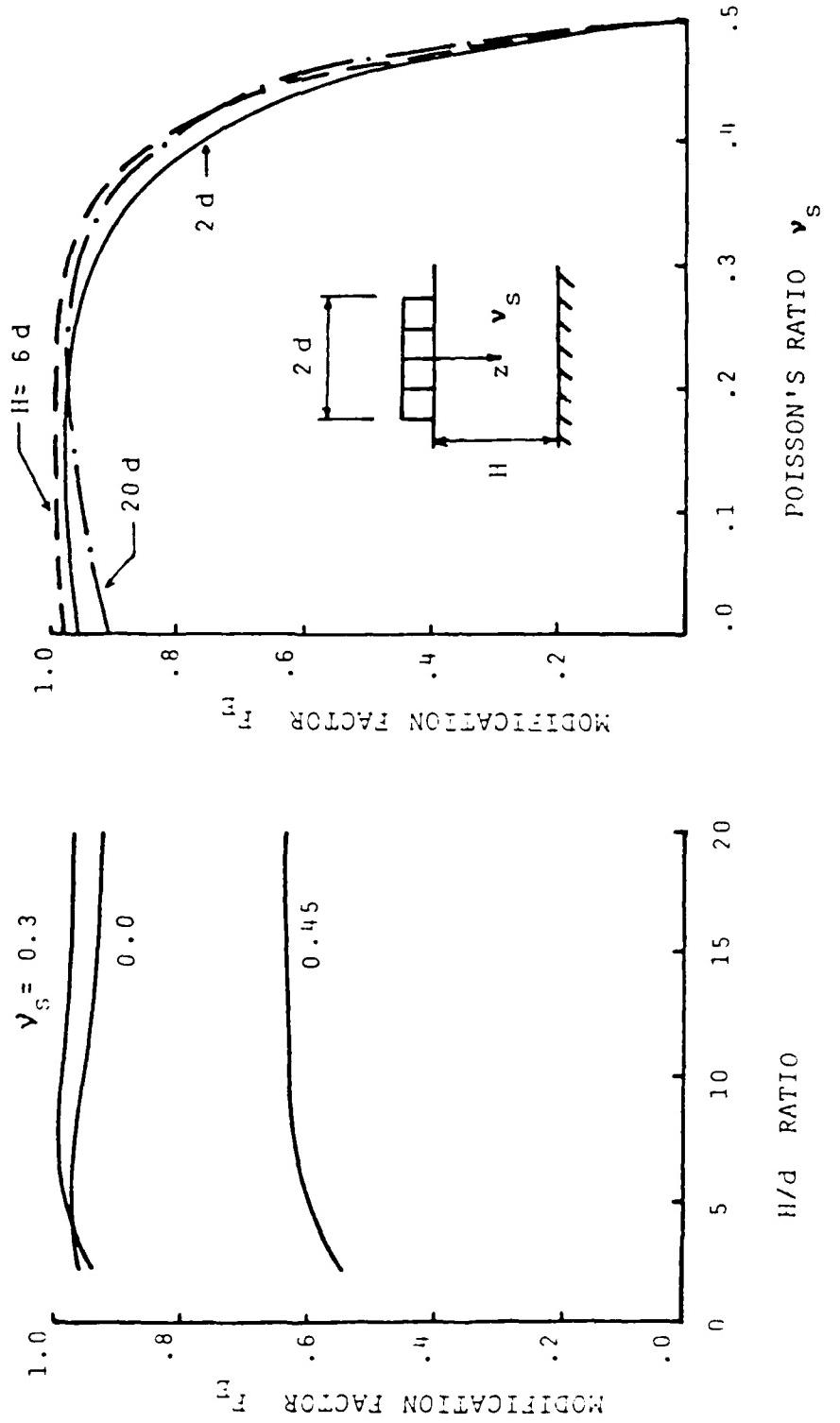
Figure 7. Modification factor F_E
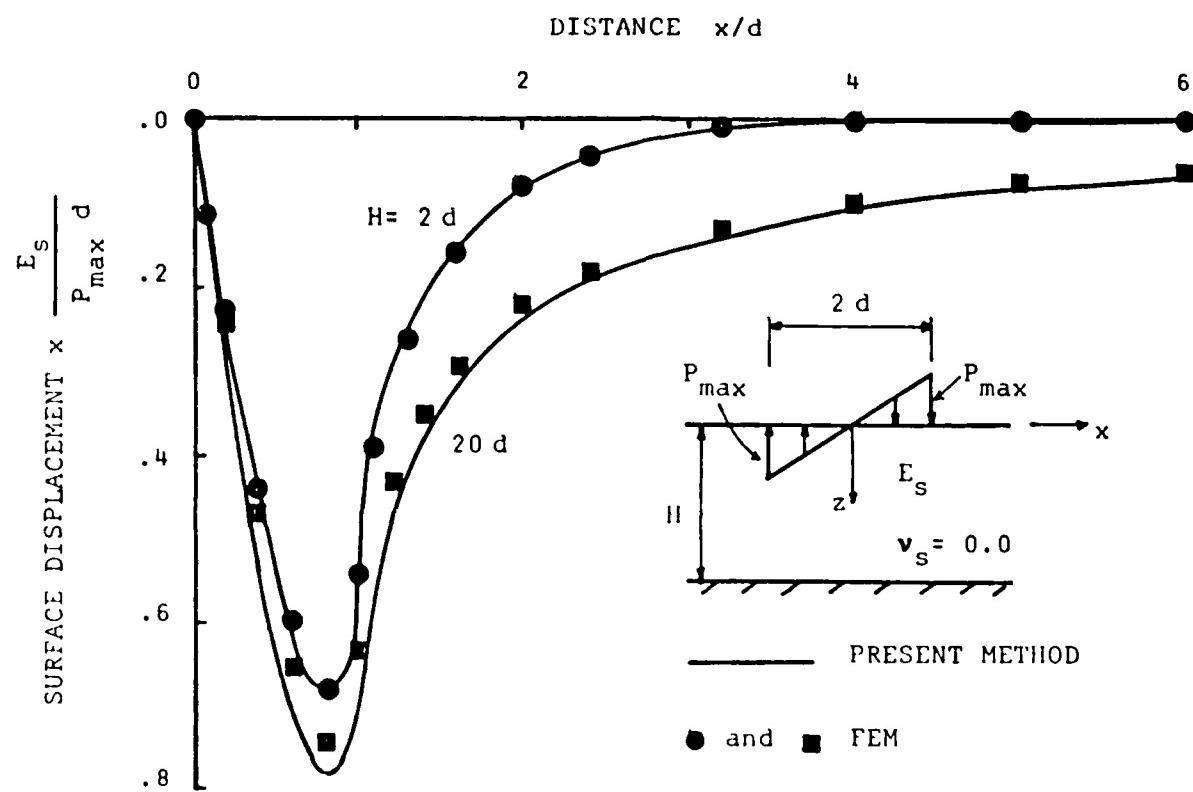
Figure 8a. Ground surface displacement computed for linearly
varying load ($\nu_s = 0.0$)
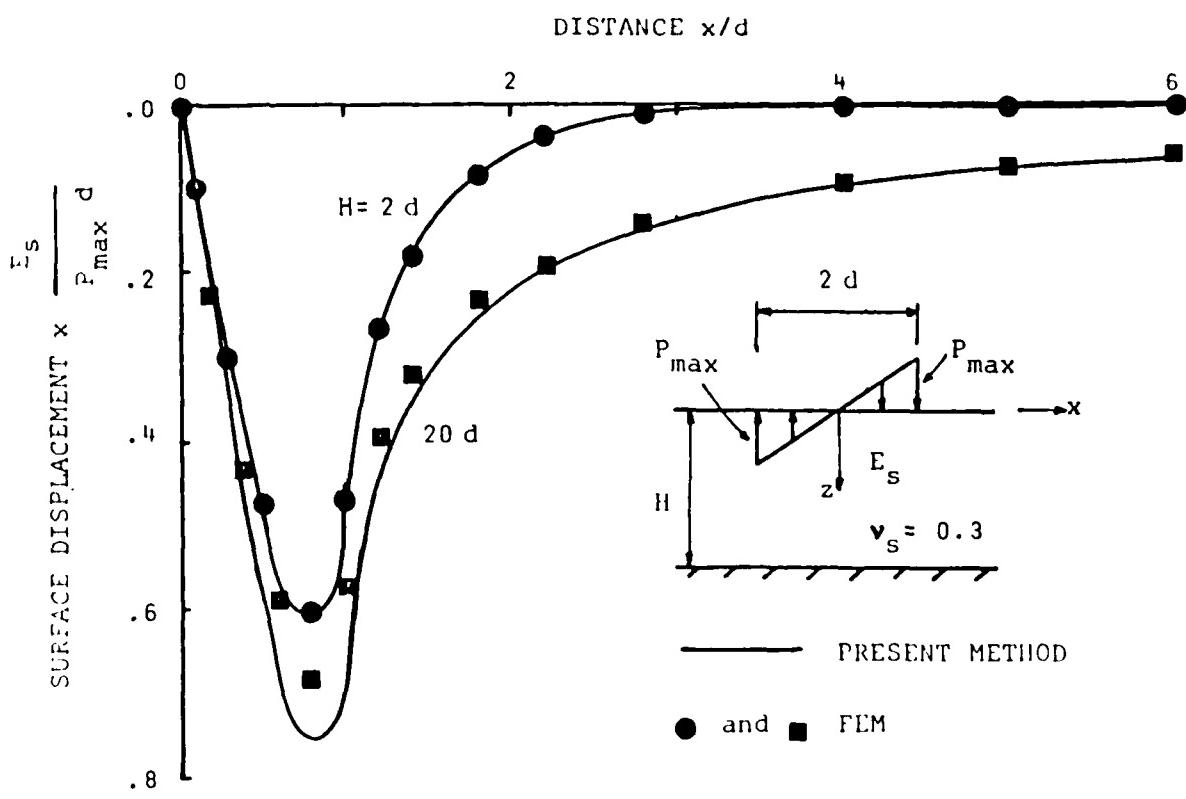
Figure 8b. Ground surface displacement computed for linearly varying load ($\nu_s = 0.3$)
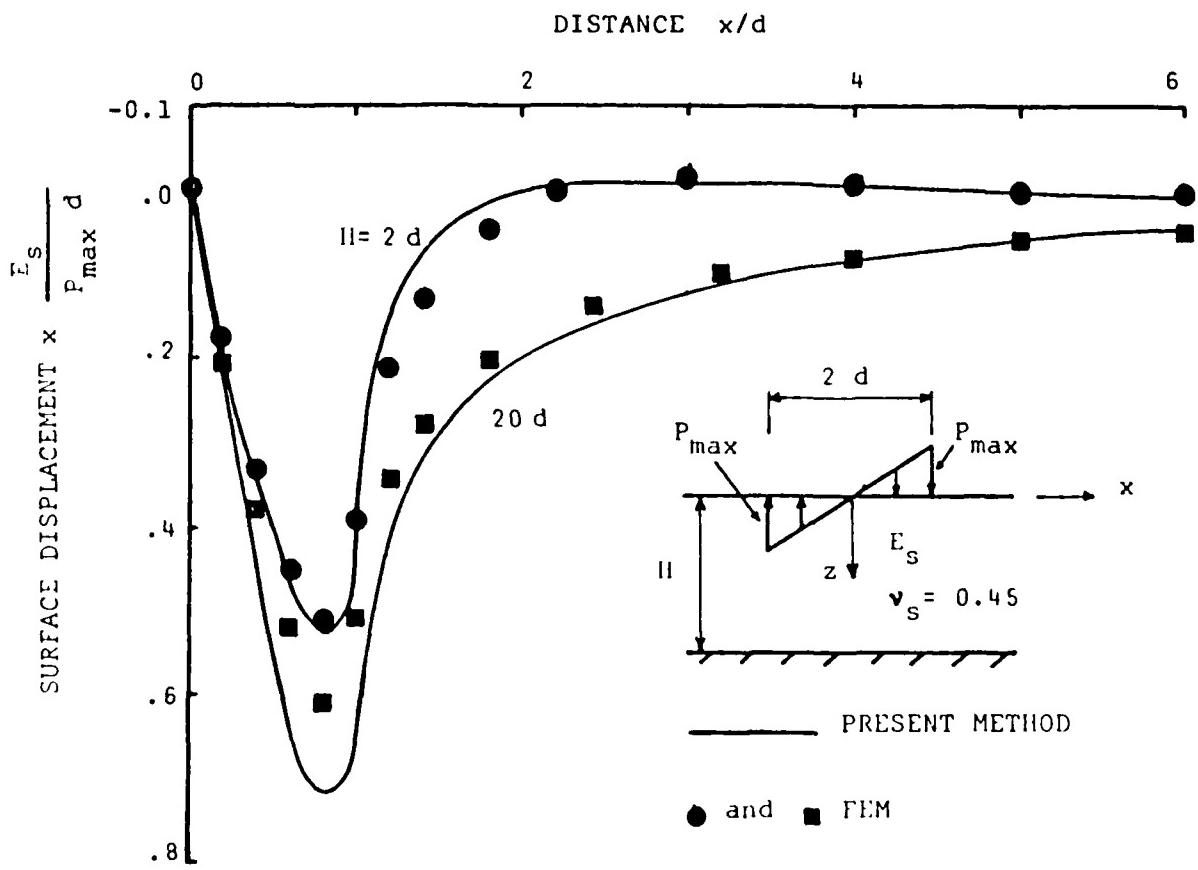
Figure 8c. Ground surface displacement computed for linearly varying load ($\nu_s = 0.45$)

or given by

$$F_E = 0.96 - 0.49\nu_s + 6.47\nu_s^2 - 15.76\nu_s^3$$

<div align="right">(49b)</div>

$$F_G = 0.5F_E$$

The factors F_E and F_G are somewhat independent of the thickness of the soil medium and loading conditions. Thus, the model parameters can be uniquely determined from equation 48 with the factors defined in equation 49, if the elastic constants of the soil medium are known.

PART V: ASSESSMENT OF ACCURACY

Determining Factors

29. The accuracy of the computed values, using the chosen model, depends on the way in which the medium is divided into a number of horizontal layers and selection of the values for the factors F_E and F_G. Figure 9 shows the surface displacements due to a uniform load applied on an elastic stratum which is equally divided into various numbers of horizontal layers for computation. The coarser the layering, the smaller is the displacement under a loaded area.
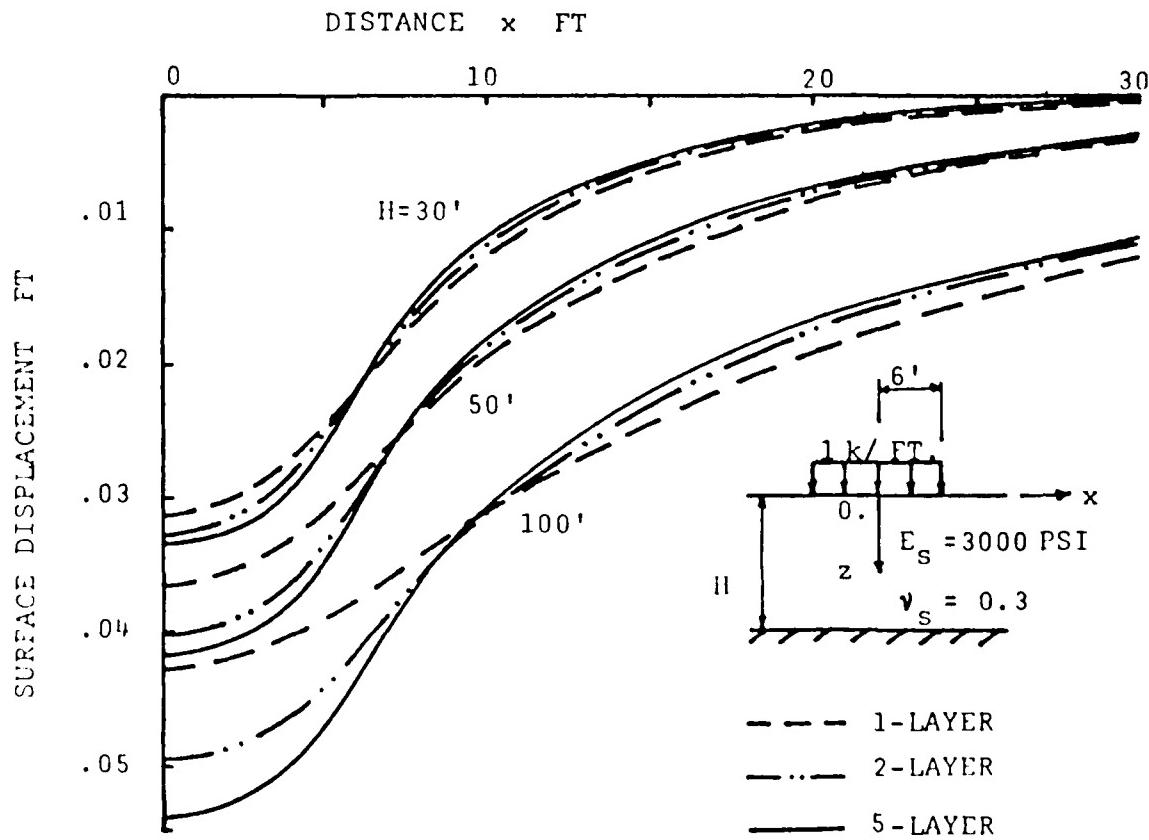
Figure 9. Ground surface displacements for various numbers of layering

30. A homogeneous soil medium is divided into a number of horizontal layers as shown in Figure 10. The surface displacements of this medium were computed for the loading and are shown in this figure. The load considered here can be produced by combining uniform and linearly varying loads that

38
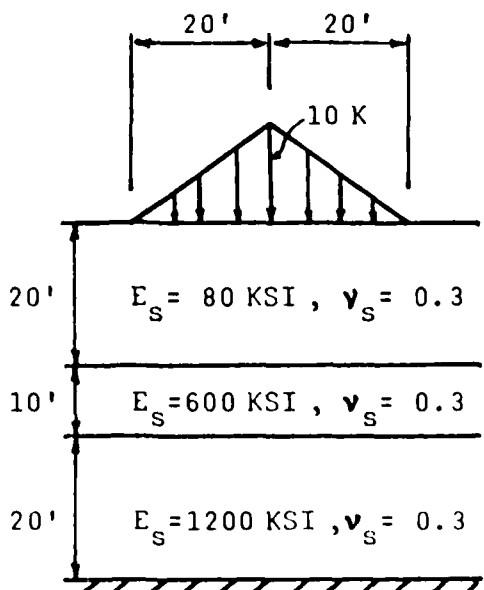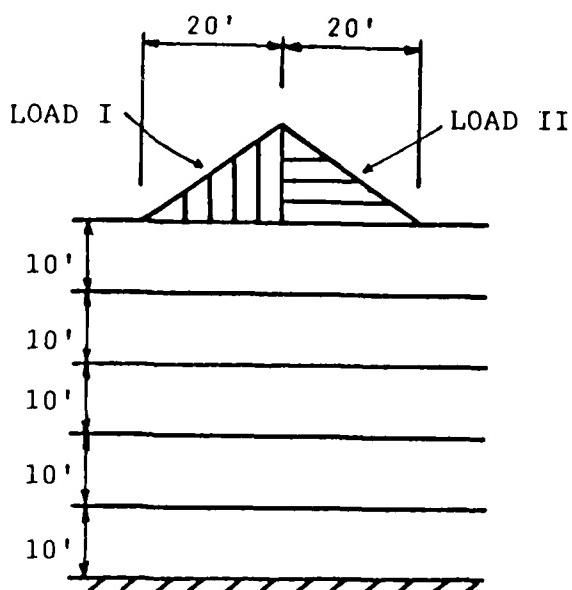
a. Case considered

b. Divided load and soil medium for computation

Figure 10. Case considered for assessment of present method

have been previously used. The factors, $F_E = 0.95$ and $F_G = 0.475$, were utilized in the analysis with the linear displacement approach. The displacements computed by this method are very close to those obtained by a FEM (Figure 11).
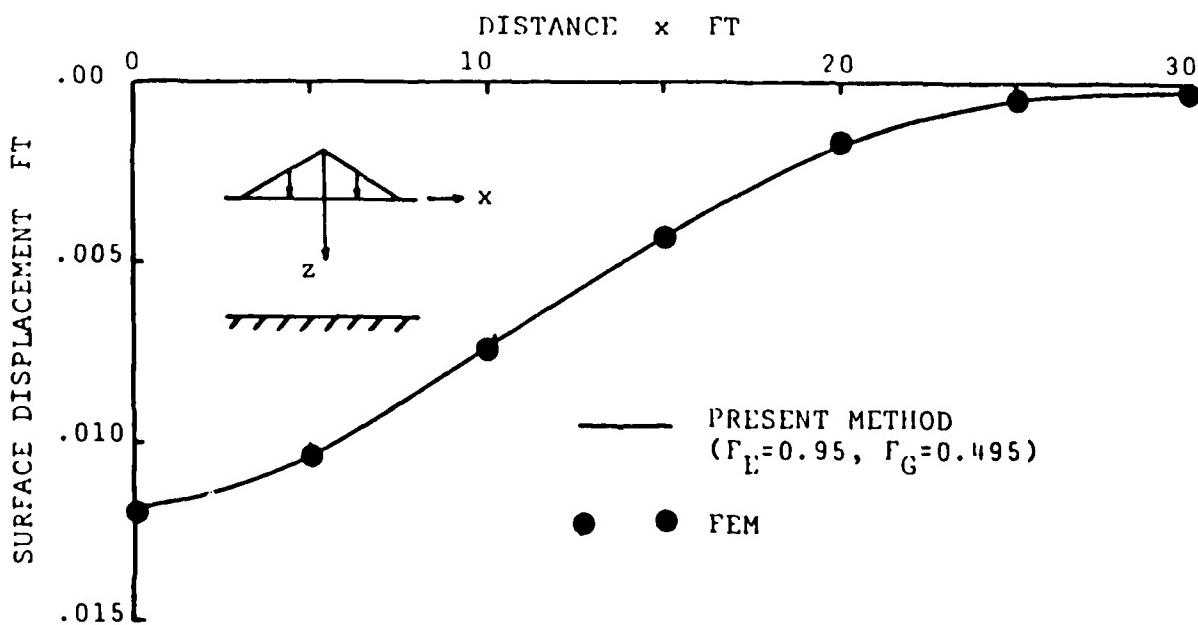
Figure 11. Ground surface displacements computed for the case shown in Figure 10

31. The displacements of a homogeneous soil medium subjected to a surface load were computed at various depths with this working model. The comparison of those computed results with those obtained by a FEM indicate that this model can produce the displacement inside the medium reasonably well as shown in Figure 12.

Model Investigation

32. The performance of the present model is investigated for the beams bearing on a homogeneous soil medium under various conditions. The conditions considered are shown in Figure 13. The factors used are $F_E = 0.95$ and $F_G = 0.475$ and the soil medium is divided into a number of horizontal layers in two different ways as shown in Figure 14. The responses of the beam and soil reactions were computed for vertical line loads, moment loads, and a uniform load applied on the beam. The computed results are plotted in Figures 15 and 16 for the loads applied symmetrically with respect to the center of the beam, in Figure 17 for the loads applied symmetrically, and in Figure 18 for a combination of various types of the load. The values obtained by a finite-element method are also plotted in the figures for a comparison. It is seen in those figures that the present model can accurately predict the beam responses even with a small number of layers.
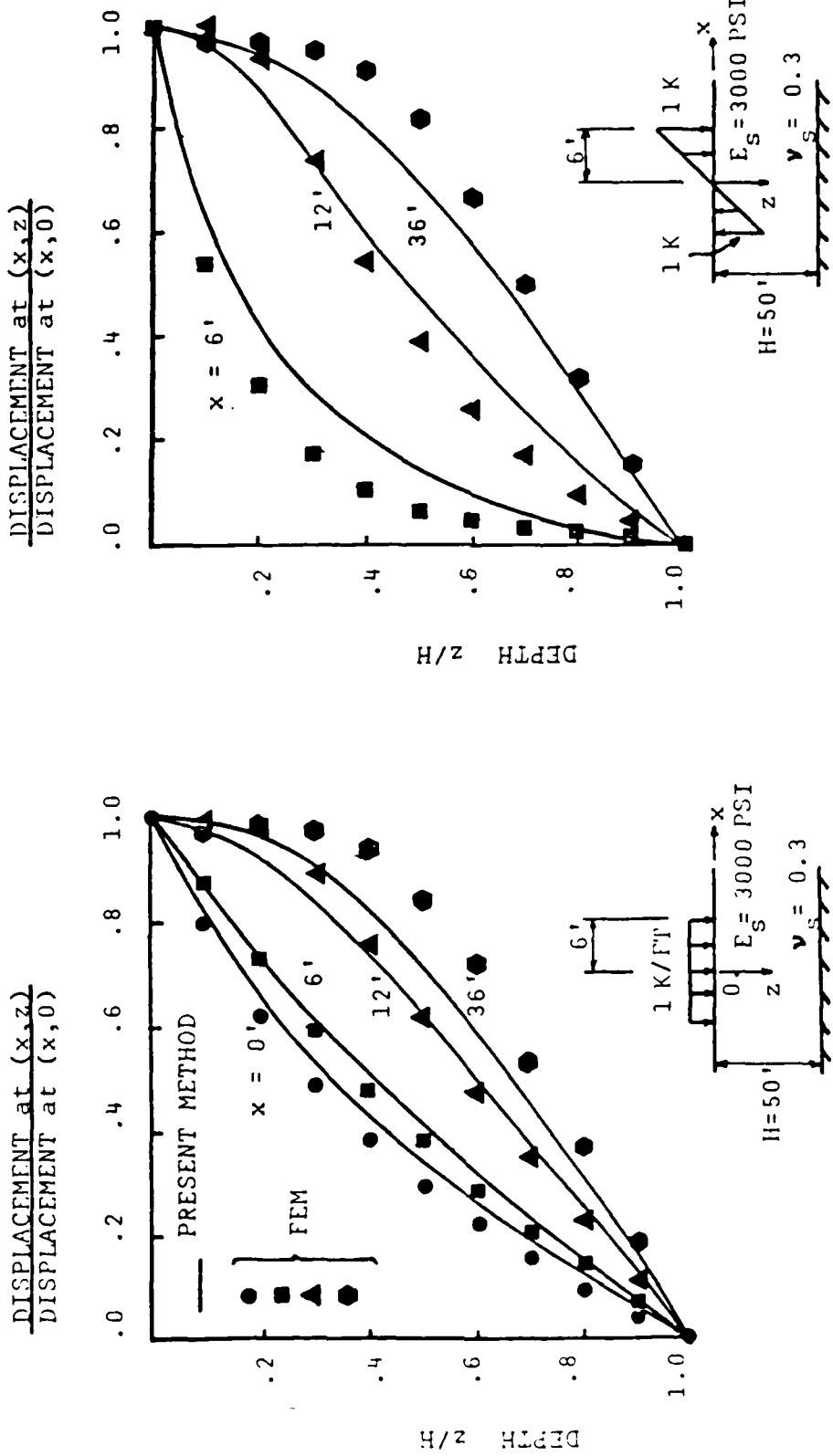
Figure 12. Computed displacement profiles with depth

41
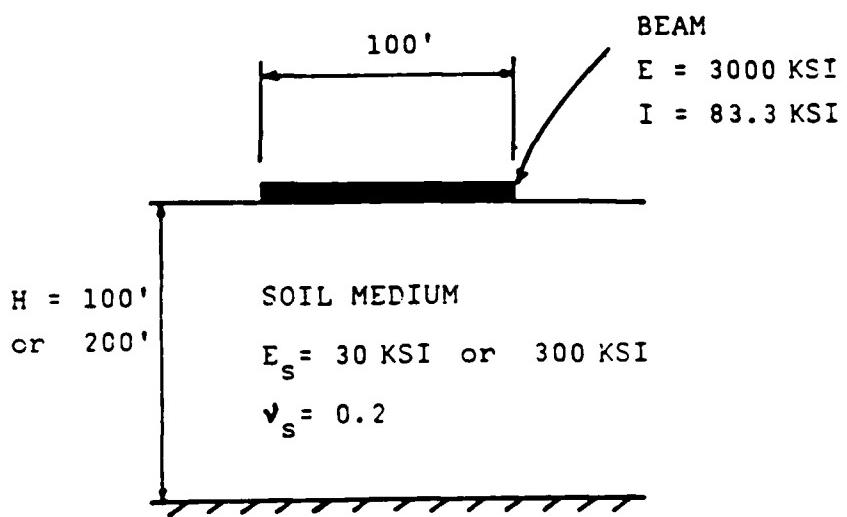
Figure 13. Case considered for assessment of
present method

H = 100' or 200'

a. Layering A   b. Layering B
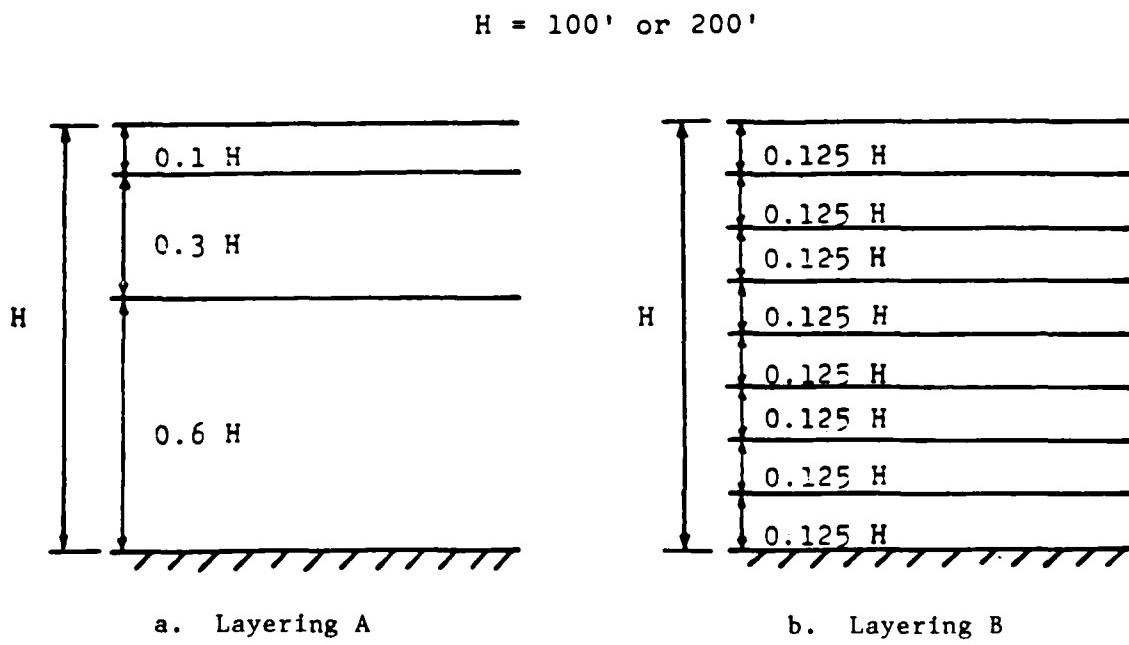
Figure 14. Soil medium divided into number of layers
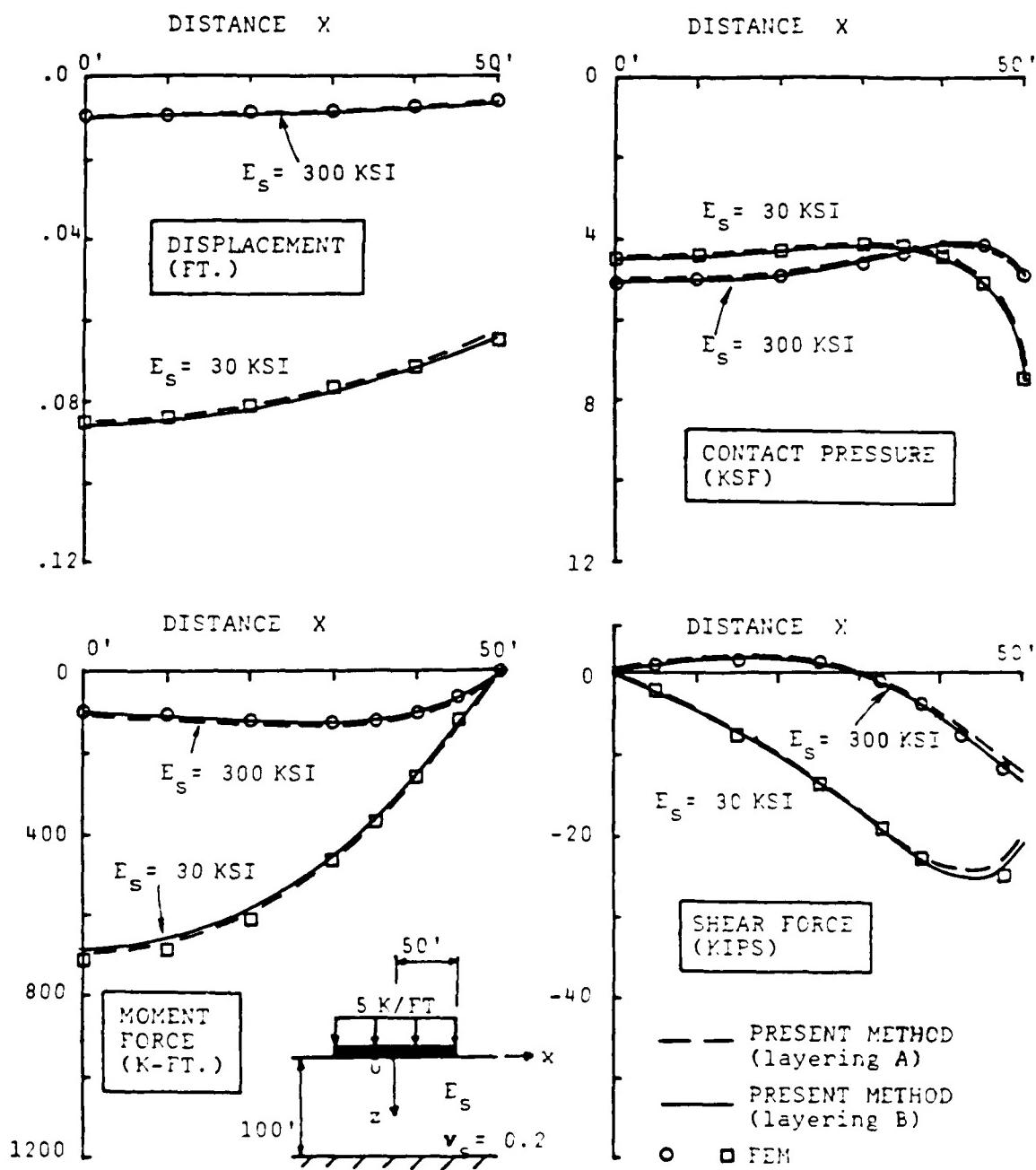
Figure 15a. Responses computed for the case considered in
Figure 13 (uniform load, H = 100')

43
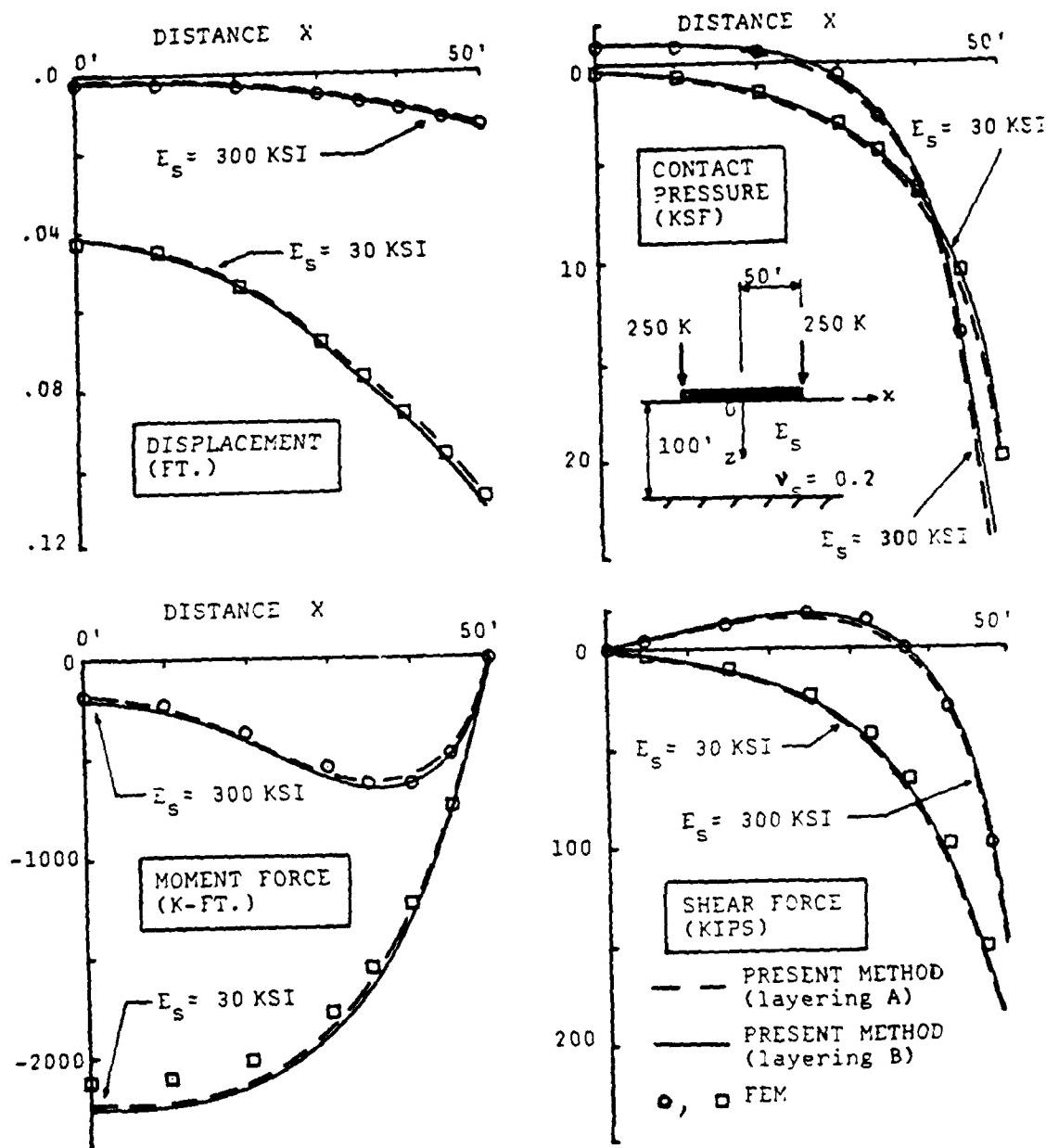
Figure 15b. Responses computed for the case considered in Figure 13 (line loads, H = 100')
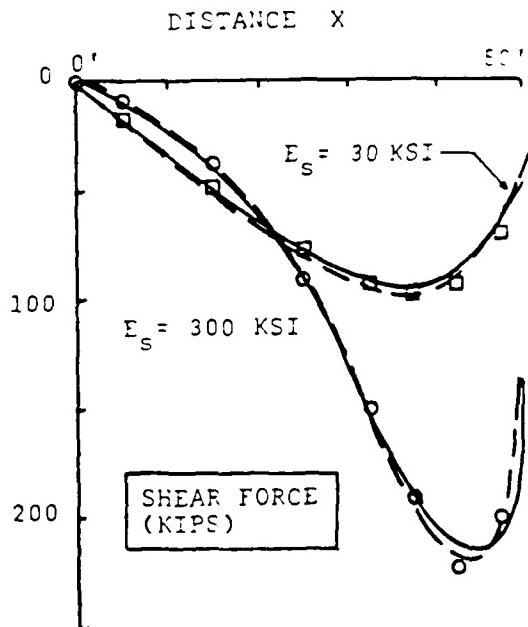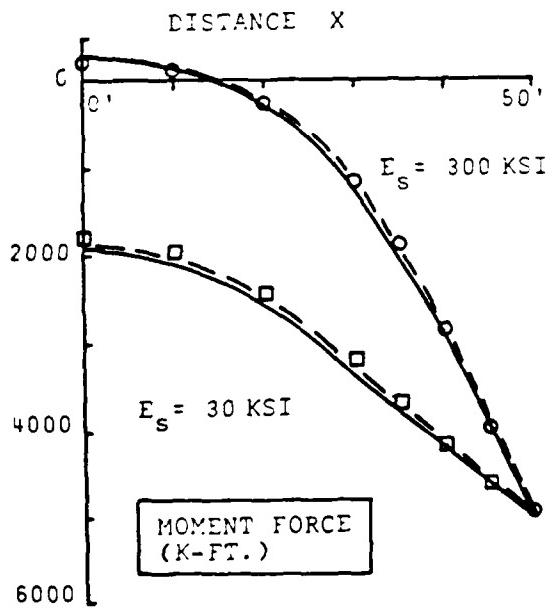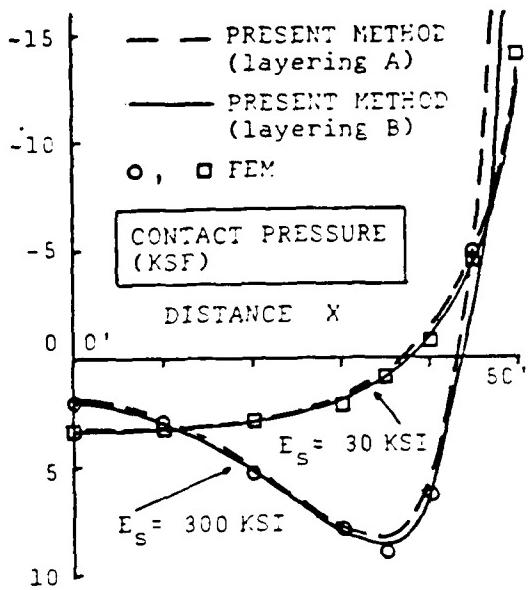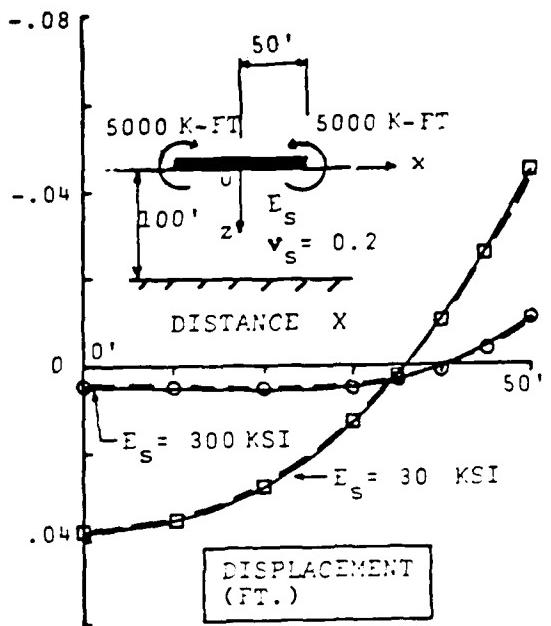
Figure 15c. Responses computed for the case considered in
Figure 13 (moment load, H = 100')

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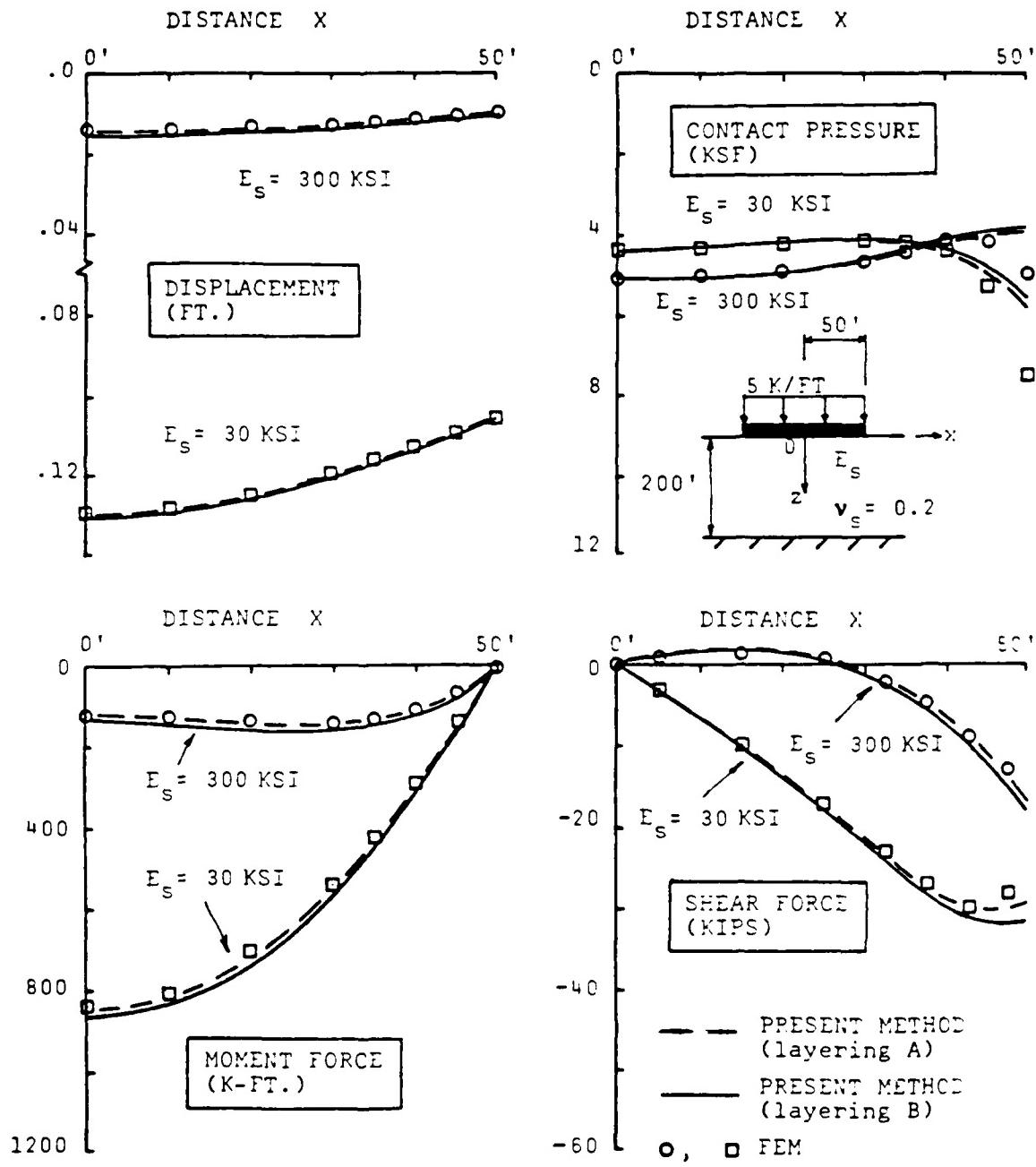
Figure 16a. Responses computed for the case considered in
Figure 13 (uniform load, H = 200')
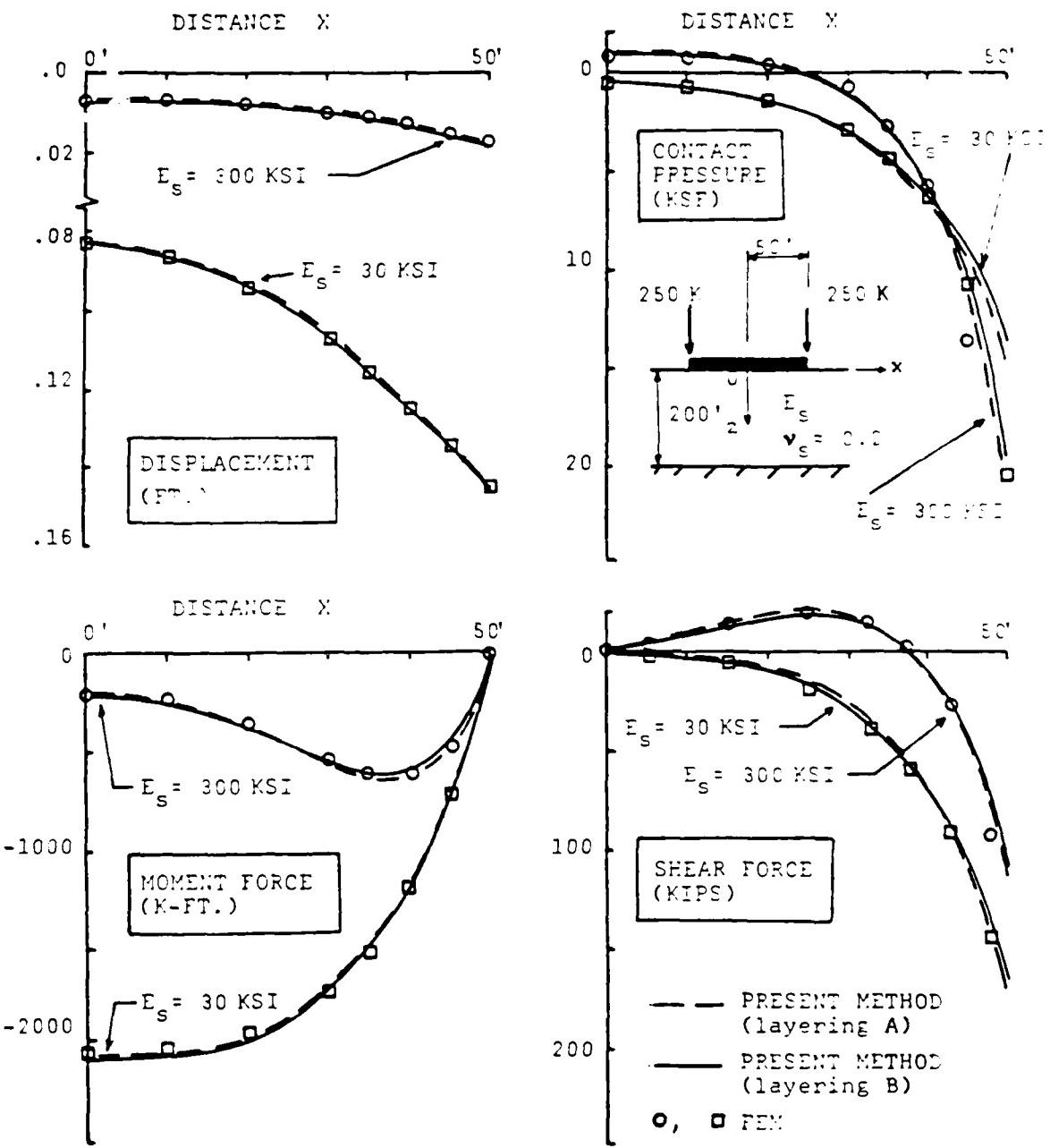
Figure 16b. Responses computed for the case considered in
Figure 13 (line loads, H = 200')
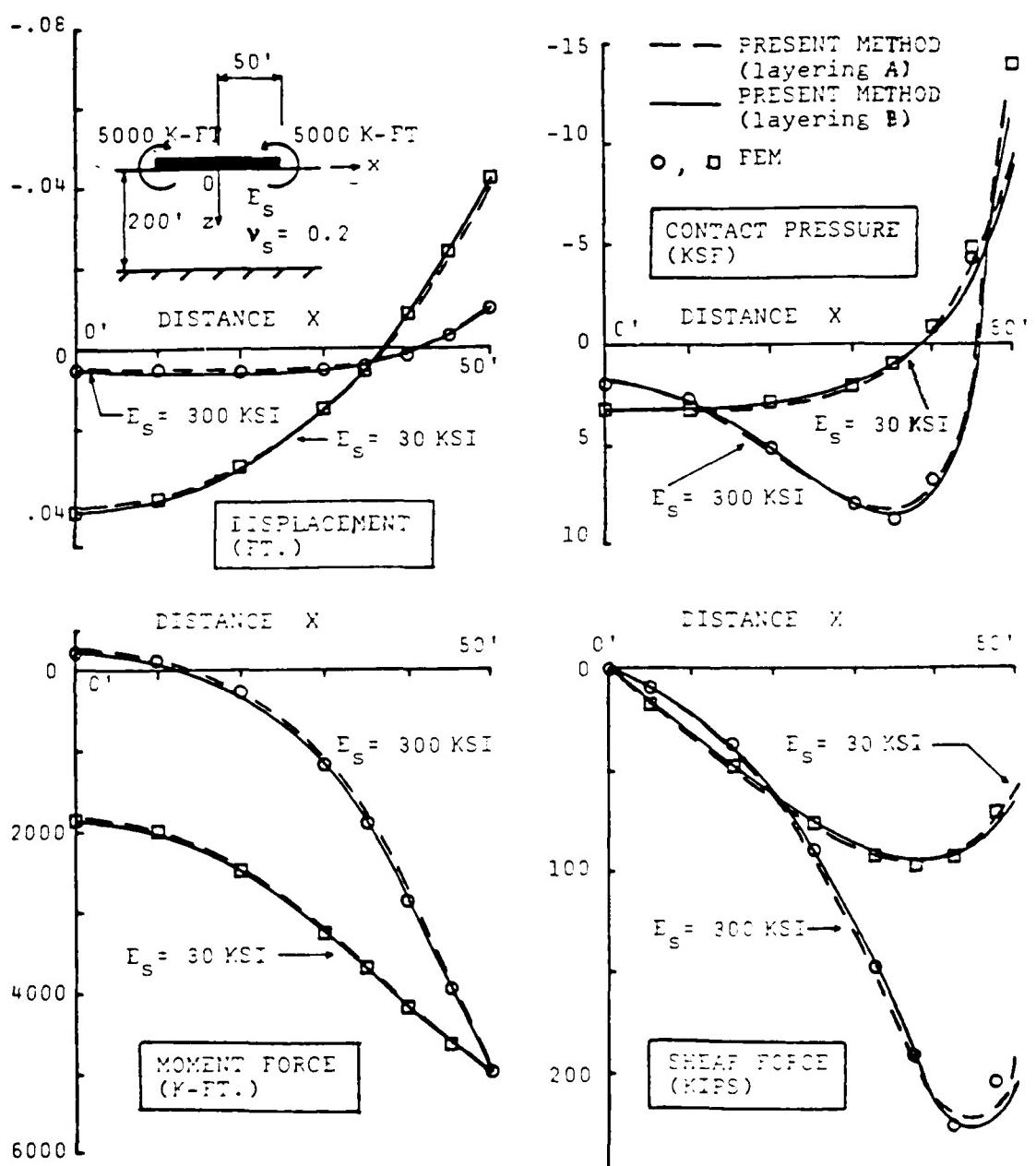
Figure 16c. Responses computed for the case considered in
Figure 13 (moment load, H = 200')

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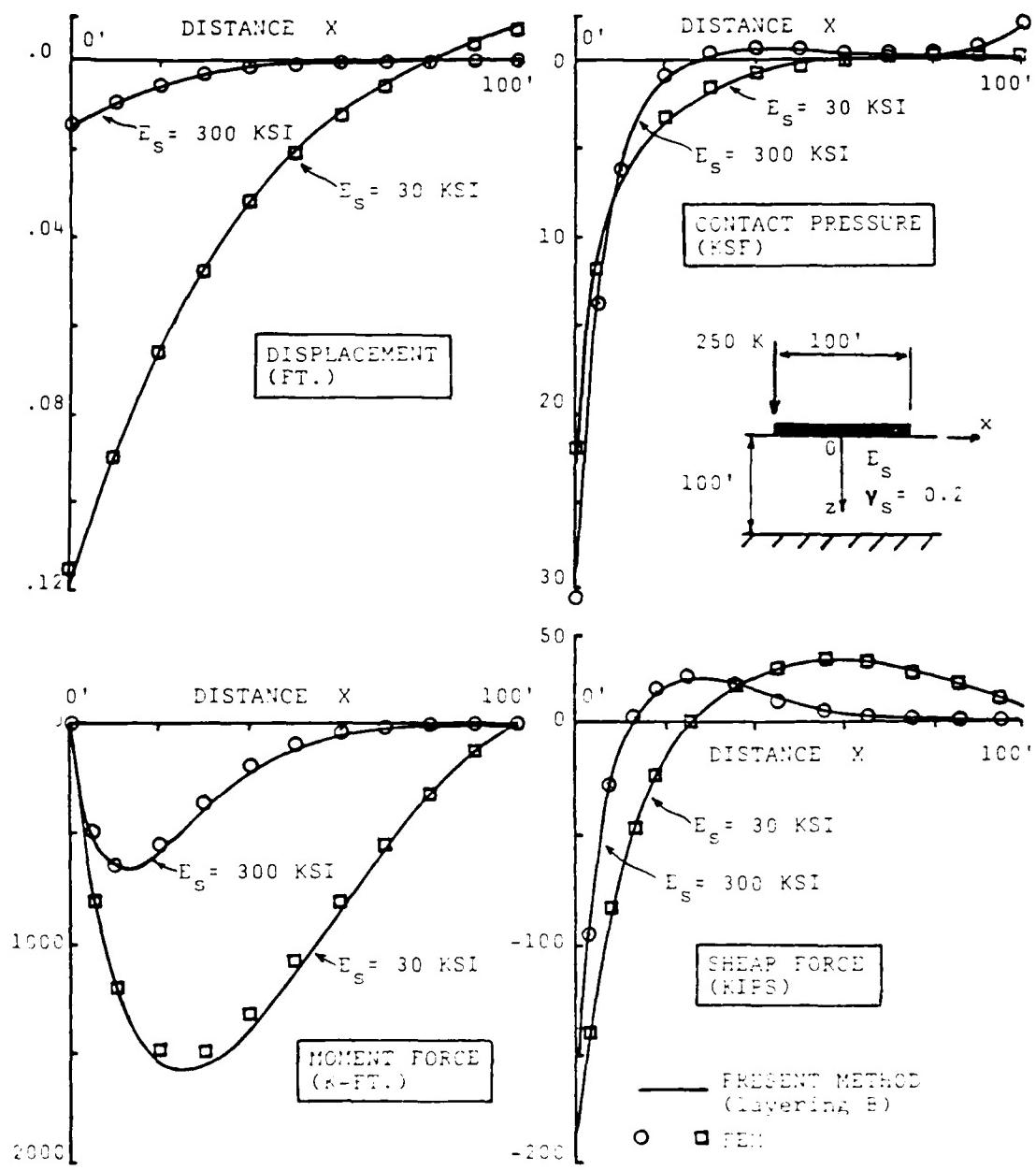
Figure 17a. Responses computed for the case considered in
Figure 13 (unsymmetric line load, H = 100')
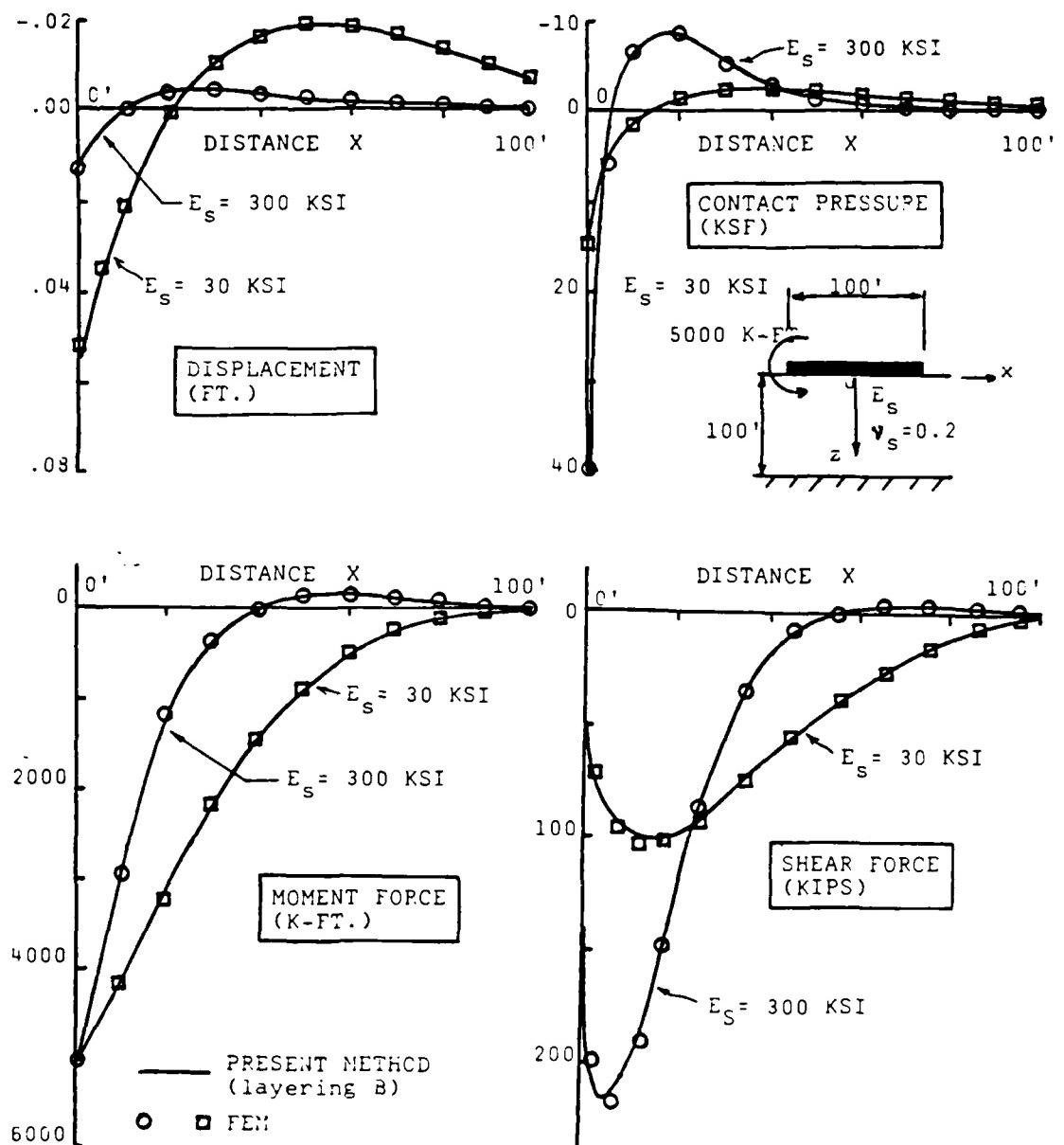
Figure 17b. Responses computed for the case considered in
Figure 13 (unsymmetric moment load, H = 100')
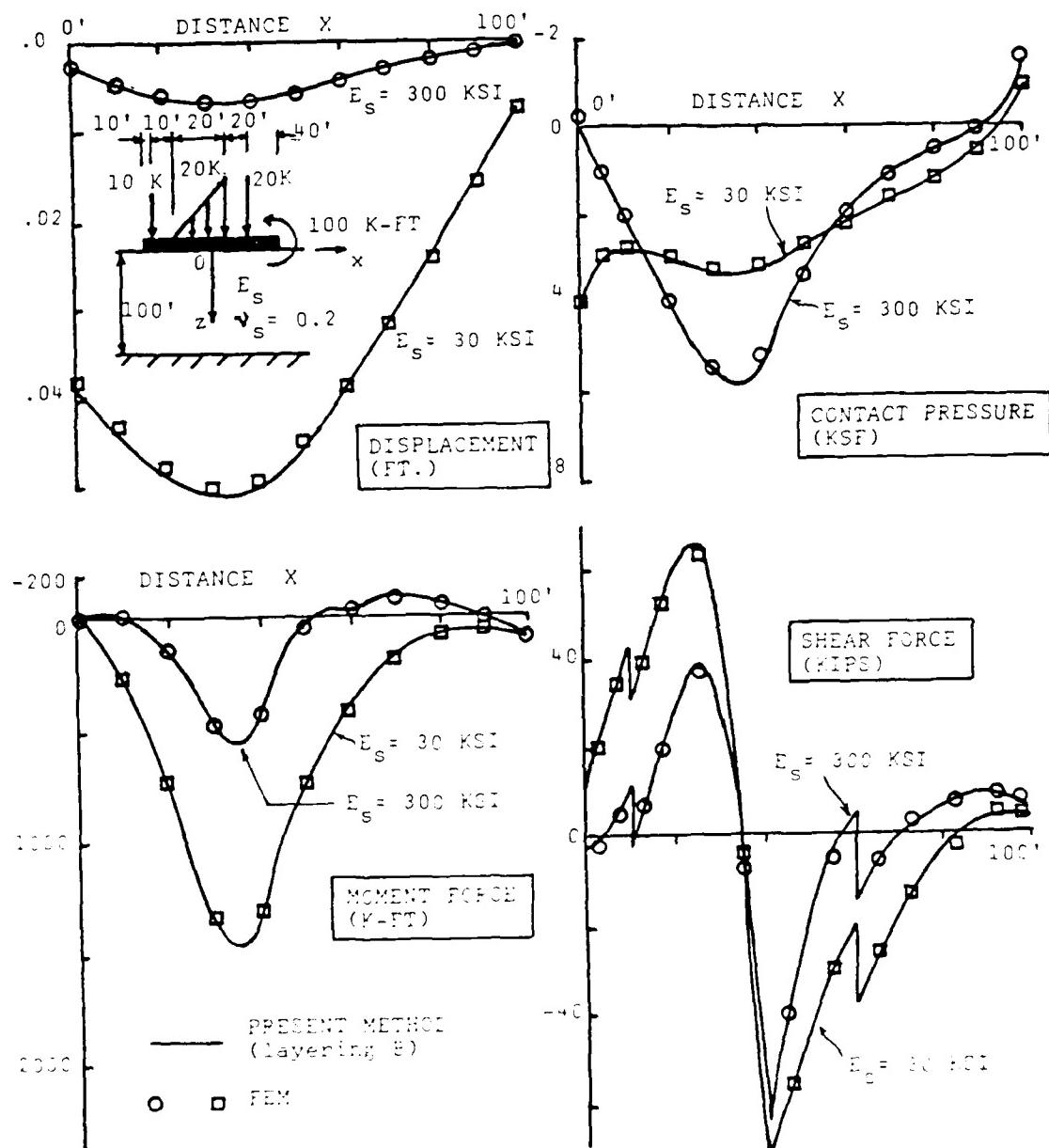
Figure 18. Responses computed for the case considered in
Figure 13 (combination of mixed loads, H = 100')

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PART VI: SUMMARIES AND CONCLUSIONS

33. This report involves the development of a two-parameter model as a layer for the analysis of the responses of a beam laying over a ground surface. This mechanical model is made of a number of vertical elastic columns defined by the two-material parameters associated with axial force in the column and friction force along the sides of the column. The material parameters are defined by the elastic constants of the medium (E_s and ν_s) and the factors F_E and F_G. The factors are rather independent of the thickness of the median and loading conditions at the surface of the medium, but dependent on Poisson's ratio. The factor F_G is found to be half of the factor $F_E (F_G = 0.5 \ F_E)$ and the values for F_E can be obtained from Figure 7 or equation 49.

34. Using the developed soil model, analytical expressions for responses of the beam, laying over a ground surface and subjected to vertical loads, were obtained. These expressions are coded in a computer program 2PARM. Performance of the soil model and the developed computer program were examined under various conditions by comparing the computed results with those obtained by a FEM. This study has confirmed that the model and computer program developed can well predict the responses of the beam bearing on the surface of an elastic homogeneous medium.

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APPENDIX A: "2PARM" PROGRAM INFORMATION

<u>Expressions for $[AB(-|x|)]$, $[AB'(-|x|)]$, $[AB''(-|x|)]$, and $[AB'''(-|x|)]$</u>

1. The matrices $[AB]$, $[AB']$, $[AB'']$, and $[AB''']$ contain $N+1$ columns with a size N in which N is the numbers of the layers in the medium. The expressions of the Nth column in these matrices are given by:

$$e^{-\alpha_n|x|} \{\cos(\beta_n|x|) \{F_n , F_n' , F_n'' \text{ or } F_n'''\}$$

$$+ \sin(\beta_n|x|) \{G_n , G_n' , G_n'' \text{ or } G_n'''\} \qquad (A1)$$

where $\alpha_{N+1} = \alpha_1$ and $\beta_{N+1} = \beta_1$. The expressions for $\{F_n\} \sim \{F_n'''\}$ and $\{G_n\} \sim \{G_n'''\}$ are:

     <u>a</u>. $[AB]$

$n < N+1$

$$\{F_n\} = \{d_n^r\}$$

$$\{G_n\} = \{d_n^i\}$$

$n = N+1$ (A2a)

$$\{F_n\} = \{d^r\}$$

$$\{G_n\} = -\{d^i\}$$

     <u>b</u>. $[AB']$

$n < N+1$

$$\{F_n'\} = \alpha_n\{d_n^r\} + \beta_n\{d_n^i\}$$

$$\{G_n'\} = -\alpha_n\{d_n^i\} - \beta_n\{d_n^r\}$$

$n = N+1$ (A2b)

$$\{F_n'\} = -\alpha_1\{d_1^i\} + \{d_1^r\}$$

$$\{G_n'\} = \alpha_1\{d_1^x\} + \beta_1\{d_1^i\}$$

A1

<u>c</u>. [AB"]

n < N+1

$$\{F_n''\} = (\alpha_n^2 + \beta_n^2) \{d_n^r\}$$

$$\{G_n''\} = -(\alpha_n + \beta_n^2) \{d_n^r\} - (\alpha_n\beta_n + \alpha_n^2) \{d_n^i\}$$

(A2c)

n = N+1

$$\{F_n''\} = -2\alpha_1\beta_1\{d_1^r\} - (\beta_1^2 - \alpha_1^2) \{d^i\}$$

$$\{G_n''\} = -(\alpha_1^2 + \beta_1^2) \{d_1^r\} - 2\alpha_1\beta_1\{d_1^i\}$$

<u>d</u>. [AB']

n < N+1

$$\{F'''\} = (\beta_n^3 - \alpha_n^3)\{d_n^r\} + (\alpha_n\beta_n^2 + \beta_n\alpha_n^2)\{d_n^i\}$$

$$\{G_n'''\} = (2\alpha_n^2\beta_n + \alpha_n\beta_n^2 + \beta_n^3)\{d_n^r\} + (\alpha_n^2\beta_n + \alpha_n^3)\{d_n^i\}$$

n = N+1

(A2d)

$$\{F_n'''\} = (3\alpha_1^2\beta_1 + \beta_1^3)\{d^r\} + (3\alpha_1\beta^2 - \alpha_1^3)\{d_1^i\}$$

$$\{G_n'''\} = (\alpha_1\beta_1^2 - \alpha_1^3)\{d_1^r\} + (\beta_1^3 - 3\alpha_1^2\beta_1)\{d_1^i\}$$

Expressions for Responses of an Infinite Beam Subjected to Line Load

2. The responses expressed in equation 38 can be written in a form of:

$$W_b(x) = P(\xi) \sum_{n=1}^{N+1} \{\cos(\beta_n|x - \xi|)F_n(1) + \sin(\beta_n|x - \xi|)G_n(1)\}r_1(n)$$

$$+ M(\xi) \sum_{n=1}^{N+1} \{\cos(\beta_n|x - \xi|)F_n(1) + \sin(\beta_n|x - \xi|)G_n(1)\}\alpha_{N+1}(n)$$

A2

$$O_b(x) = P(\xi) \sum_{n=1}^{N+1} \{\cos(\beta_n|x - \xi|)F_n'(1) + \sin(\beta_n|x - \xi|)G_n^1(1)\}\alpha_1(n)$$

$$+ M(\xi) \sum_{n=1}^{N+1} \{\cos(\beta_n|x - \xi|)F_n'(1) + \sin(\beta_n|x - \xi|G_n^1(1)\}\alpha_{N+1}(n)$$

$$M_b(x) = -EI-P(\xi) \sum_{n=1}^{N+1} \{\cos(\beta_n|x - \xi|)F_n''(1) + \sin(\beta_n|x - \xi|)G_n''(1)\}\alpha_1(n)$$

$$+ EI-M(\xi) \sum_{n=1}^{N+1} \{\cos(\beta_n|x - \xi|)F_n''(1) + \sin(\beta_n|x - \xi|)G_n''(1)\}\alpha_{N+1}(n)$$

$$T_b(x) = -EI\ P(\xi) \sum_{n=1}^{N+1} \{\cos(\beta_n|x - \xi|)F_n'''(1) + \sin(\beta_n|x - \xi|)G_n'''(1)\}\alpha_1(n)$$

$$- EI-M(\xi) \sum_{n=1}^{N+1} \{\cos(\beta_n|x - \xi|)F_n'''(1) + \sin(\beta_n|x - \xi|)G_n'''(1)\}\alpha_{N+1}(n)$$

$$(A3)$$

where $F_n(1) \sim F_n'''(1)$ and $G_n(1) \sim G_n'''(1)$ are the numbers located at the first position in the vectors given in equation B2.

Expressions for $[D_a] \sim [D_a''']$ and $[D_b] \sim [D_a''']$

3. The matrices $[D_a]$, $[D_a']$, $[D_a'']$, and $[D_a''']$ contain N+1 columns with a size N in which N is the numbers of layers in the medium. The expressions of the Nth column in these matrices are expressed by:

A3

$$\{C_n\} + \sum_{z=1}^{2} e^{-\alpha_n |x - x_j|} (-1)^j \{\cos (\beta_n |x - x_j|)\{H_n\}$$

$$\text{(A4)}$$

$$+ \sin (\beta_n |x - x_j|)\{I_n\}\}$$

where $\alpha_{N+1} = \alpha_1$ and $\beta_{N+1} = \beta_1$. $\{H_n\}$, $\{Z_n\}$, and $\{C_n\}$ are vectors expressed by:

$\underline{a}$. $[D_a]$

$x \leq x_1$, $n < N+1$

$$\{H_n\} = \left[- \frac{x_j \alpha_n}{\alpha_n^2 + \beta_n^2} + \frac{\alpha_n^2 - \beta_n^2}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\} + \left[\frac{x_j \beta_n}{\alpha_n^2 + \beta_n^2} - \frac{2\alpha_n \beta_n}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{I_n\} = \left[- \frac{x_j \beta_n}{\alpha_n^2 + \beta_n^2} - \frac{2\alpha_n \beta_n}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\} + \left[\frac{x_j \alpha_n}{\alpha_n^2 + \beta_n^2} - \frac{\alpha_n^2 - \beta_n^2}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{C_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \left[- \frac{x_j \beta_1}{\alpha_1^2 + \beta_1^2} - \frac{2\alpha_1 \beta_1}{(\alpha_1^2 + \beta_1^2)^2} \right]\{d_n^r\} + \left[- \frac{x_j \alpha_1}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1^2 - \beta_1^2}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{I_n\} = \left[- \frac{2_j \alpha_1}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1^2 - \beta_1^2}{(\alpha_1^2 + \beta_1^2)^2} \right]\{d_n^r\} + \left[\frac{x_j \alpha_1}{\alpha_1^2 + \beta_1^2} - \frac{2\alpha_1 \beta_1}{(\alpha_1^2 + \beta_1^2)^2} \right]\{d_n^i\}$$

$$\{C_n\} = \{0\}$$

$$x_1 \leq x \leq x_2 \; , \; n < N+1$$

$$\{H_n\} = (-1)^z \left[\frac{x_j \alpha_n}{\alpha_n^2 + \beta_n^2} - \frac{\alpha_n^2 - \beta_n^2}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^r\} + \left[-\frac{x_j \beta_n}{\alpha_n^2 + \beta_n^2} + \frac{2\alpha_n \beta_n}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^i\}$$

$$\{I_n\} = (-1)^z \left[\frac{x_j \beta_n}{\alpha_n^2 + \beta_n^2} + \frac{2\alpha_n \beta_n}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^r\} + \left[-\frac{x_j \alpha_n}{\alpha_n^2 + \beta_n^2} + \frac{\alpha_n^2 - \beta_n^2}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^i\}$$

$$\{C_n\} = \frac{2}{(\alpha_n^2 + \beta_n^2)^2} \left[-\beta_n x + \frac{2\alpha_n \beta_n}{\alpha_n^2 + \beta_n^2} \right] \{d_n^r\}$$

$$n = N+1$$

$$\{H_n\} = \left[\frac{x_j \beta_1}{\alpha_1^2 + \beta_1^2} - \frac{2\alpha_1 \beta_1}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^r\} + (-1)^j \left[\frac{x_j - \alpha_1}{\alpha_1^2 + \beta_1^2} - \frac{\alpha_1^2 - \beta_1^2}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^i\}$$

$$\{I_n\} = \left[\frac{x_j \alpha_1}{\alpha_1^2 + \beta_1^2} - \frac{\alpha_1^2 - \beta_1^2}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^r\} + (-1)^j \left[-\frac{x_j \beta_1}{\alpha_1^2 + \beta_1^2} + \frac{2\alpha_1 \beta_1}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^i\}$$

$$\{C_n\} = \frac{2}{(\alpha_1^2 + \beta_1^2)^2} \left[-\beta_1 x - \frac{2\alpha_1 \beta_1}{\alpha_1^2 + \beta_1^2} \right] \{d_n^r\}$$

A5

$x \geq x_2$, $n < N+1$

$$\{H_n\} = \left[-\frac{x_j \alpha_n}{\alpha_n^2 + \beta_n^2} + \frac{\alpha_n^2 - \beta_n^2}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\} + \left[-\frac{x_j \beta_n}{\alpha_n^2 + \beta_n^2} + \frac{2\alpha_n \beta_n}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{I_n\} = \left[\frac{x_j \beta_n}{\alpha_n^2 + \beta_n^2} - \frac{2\alpha_n \beta_n}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\} + \left[-\frac{x_j \alpha_n}{\alpha_n^2 + \beta_n^2} + \frac{\alpha_n^2 - \beta_n^2}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{C_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \left[\frac{x_j \beta_1}{\alpha_1^2 + \beta_1^2} - \frac{2\alpha_1 \beta_1}{(\alpha_1^2 + \beta_1^2)^2} \right]\{d_n^r\} + \left[-\frac{x_j \alpha_1}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1^2 - \beta_1^2}{\alpha_1^2 + \beta_1^2} \right]\{d_n^i\}$$

$$\{I_n\} = \left[\frac{x_j \alpha_1}{\alpha_1^2 + \beta_1^2} - \frac{\alpha_1^2 - \beta_1^2}{(\alpha_1^2 + \beta_1^2)^2} \right]\{d_n^r\} + \left[\frac{x_j \beta_1}{\alpha_1^2 + \beta_1^2} - \frac{2\alpha_1 \beta_1}{(\alpha_1^2 + \beta_1^2)^2} \right]\{d_n^i\}$$

$$\{C_n\} = \{0\}$$

b. $[D_a']$

$x \leq x_1$, $n < N+1$

$$\{H_n\} = \left[x_j - \frac{\alpha_n(\alpha_n - \beta_n)}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\} + \left[-\frac{2x_j \alpha_n \beta_n}{\alpha_n^2 + \beta_n^2} + \frac{\beta_n(3\alpha_n^2 + \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{I_n\} = \left[-\frac{2x_j \alpha_n \beta_n}{\alpha_n^2 + \beta_n^2} + \frac{\beta_n(\alpha_n^2 + \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\} + \left[-\frac{x_j(\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} + \frac{\alpha_n(\alpha_n^2 - \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{C_n\} = \{0\}$$

A6

$n = N+1$

$$\{H_n\} = \left[\frac{2x_j\alpha_1\beta_1}{\alpha_1^2 + \beta_1^2} - \frac{\beta_1(3\alpha_1^2 + \beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2}\right]\{d_n^r\} + \left[x_j - \frac{\alpha_1(\alpha_1^2 - \beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2}\right]\{d_n^i\}$$

$$\{I_n\} = \left[\frac{x_j(\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} - \frac{\alpha_1(\alpha_1^2 - \beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2}\right]\{d_n^r\} + \left[\frac{2x_j\alpha_1\beta_1}{\alpha_1^2 + \beta_1^2} + \frac{\beta_1(\alpha_1^2 + \beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2}\right]\{d_n^i\}$$

$$\{C_n\} = \{0\}$$

$x_1 \leq x \leq x_2$, $n < N+1$

$$\{H_n\} = (-1)^j \left[-x_j + \frac{\alpha_n}{\alpha_n^2 + \beta_n^2}\right]\{d_n^r\} + \frac{-\beta_n}{\alpha_n^2 + \beta_n^2}\{d_n^i\}$$

$$\{I_n\} = (-1)^j \frac{-\beta_n}{\alpha_n^2 + \beta_n^2}\{d_n^r\} + \left[x_j - \frac{\alpha_n}{\alpha_n^2 + \beta_n^2}\right]\{d_n^i\}$$

$$\{C_n\} = \frac{-2\beta_n}{(\alpha_n^2 + \beta_n^2)^2}\{d_n^i\}$$

$n = N+1$

$$\{H_n\} = \frac{\beta_1}{\alpha_1^2 + \beta_1^2}\{d_n^r\} + (-1)^j\left[-x_j + \frac{\alpha_1}{\alpha_1^2 + \beta_1^2}\right]\{d_n^i\}$$

$$\{I_n\} = \left[-x_j + \frac{\alpha_1}{\alpha_1^2 + \beta_1^2}\right]\{d_n^r\} + (-1)^j \frac{-\beta_1}{\alpha_1^2 + \beta_1^2}\{d_n^i\}$$

$$\{C_n\} = \frac{2\beta_n}{(\alpha_1^2 + \beta_1^2)^2}\{d_n^r\}$$

A7

$x \geq x_2$, $n < N+1$

$$\{H_n\} = \left[x_j - \frac{\alpha_n(\alpha_n^2 - \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\} - \left[-\frac{2\alpha_n\beta_n x_j}{\alpha_n^2 + \beta_n^2} + \frac{\beta_n(3\alpha_n^2 + \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \{d_n^i\} \right]$$

$$\{I_n\} = \left[\frac{-2\alpha_n\beta_n x_j}{\alpha_n^2 + \beta_n^2} + \frac{\beta_n}{\alpha_n^2 + \beta_n^2} \right]\{d_n^r\} - \left[-\frac{x_j(\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} + \frac{\alpha_n(\alpha_n^2 - \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{C_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \left[-\frac{2\alpha_1\beta_1 x_j}{\alpha_1^2 + \beta_1^2} + \frac{\beta_1(3\alpha_1^2 + \beta_1^2)}{\alpha_1^2 + \beta_1^2} \right]\{d_n^r\} + \left[x_j - \frac{\alpha_1(\alpha_1^2 - \beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2} \right]\{d_n^i\}$$

$$\{I_n\} = \left[-\frac{x_j(\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1(\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} \right]\{d_n^r\} + \left[-\frac{2x_j\alpha_1\beta_1}{\alpha_1^2 + \beta_1^2} + \frac{\beta_1}{\alpha_1^2 + \beta_1^2} \right]\{d_n^i\}$$

$$\{C_n\} = \{0\}$$

$\underline{c}$. $[D_a'']$

$x < x_1$, $n < N+1$

$$\{H_n\} = \left[\frac{-x_j\alpha_n(\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} - \frac{2\alpha_n^2\beta_n^2 - \alpha_n^4 + \beta_n^4}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\}$$

$$+ \left[\frac{x_j\beta_n(3\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} - \frac{4\alpha_n^3\beta_n}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

A8

$n < N+1$

$$\{I_n\} = \left[\frac{x_j \beta_n (3\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} + \frac{2\alpha_n \beta_n (\beta_n^2 - \alpha_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{-x_j \alpha_n (\alpha_n^2 - 3\beta_n^2)}{\alpha_n^2 + \beta_n^2} + \frac{4\alpha_n^2 \beta_n^2 - \alpha_n^4 - \beta_n^4}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \left[\frac{x_j \beta_1 (\beta_1^2 - 3\alpha_1^2)}{\alpha_1^2 + \beta_1^2} + \frac{4\alpha_1^3 \beta_1}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{-x_j \alpha_1 (\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} - \frac{2\alpha_1^2 \beta_1^2 - \alpha_1^4 + \beta_1^4}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^i\}$$

$$\{I_n\} = \left[\frac{x_j \alpha_1 (3\beta_1^2 - \alpha_1^2)}{\alpha_1^2 + \beta_1^2} - \frac{4\alpha_1^2 \beta_1^2 - \alpha_1^4 + \beta_1^4}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{x_j \beta_1 (3\beta_1^2 + \alpha_1^2)}{\alpha_1^2 + \beta_1^2} + \frac{2\alpha_1 \beta_1 (\beta_1^2 - \alpha_1^2)}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$x_1 \leq x \leq x_2$, $n < N+1$

$$\{H_n\} = (-1)^j [x_{jn} - 1] \{d_n^r\} + [x_{jn}]\{d_n^i\}$$

$$\{I_n\} = (-1)^j [x_{jn}] \{d_n^r\} + [1 - x_{jn}]\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = [-x_j \beta_1]\{d_n^r\} + (-1)^j [x_j \alpha_1 - 1]\{d_n^i\}$$

$$\{I_n\} = [x_j \alpha_1 - 1]\{d_n^r\} + (-1)^j [x_j \beta_1]\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$x > x_2$, $n < N+1$

$$\{H_n\} = \left[\frac{x_j \alpha_n (\beta_n^2 - \alpha_n^2)}{\alpha_n^2 + \beta_n^2} - \frac{2\alpha_n^2 \beta_n^2 - \alpha_n^4 + \beta_n^4}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\}$$

$$+ \left[\frac{-x_{jn}(3\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} - \frac{4\alpha_n^3 \beta_n}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{I_n\} = \left[\frac{x_j \beta_n (3\alpha_n^2 + \beta_n^2)}{\alpha_n^2 + \beta_n^2} - \frac{2\alpha_n \beta_n (\alpha_n^2 - \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^r\}$$

$$+ \left[\frac{x_j \alpha_n (3\beta_n^2 - \alpha_n^3)}{\alpha_n^2 + \beta_n^2} + \frac{4\alpha_n^2 \beta_n^2 - \alpha_n^4 - \beta_n^4}{(\alpha_n^2 + \beta_n^2)^2} \right]\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

A10

$n = N+1$

$$\{H_n\} = \left[\frac{x_j \beta_1 (3\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} + \frac{4\alpha_1^3 \beta_1}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{-x_j \alpha_1 (\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} - \frac{2\alpha_1^2 \beta_1^2 - \alpha_1^4 + \beta_1^4}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^i\}$$

$$\{I_n\} = \left[\frac{x_j \alpha_1 (\alpha_1^2 - 3\beta_1^2)}{\alpha_1^2 + \beta_1^2} - \frac{4\alpha_1^2 \beta_1^2 - \alpha_1^4 + \beta_1^4}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{x_j \beta_1 (3\alpha_1^2 + \beta_1^2)}{\alpha_1^2 + \beta_1^2} + \frac{2\alpha_1 \beta_1 (\alpha_1^2 - \beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$\underline{d}.$ $[D_a''']$

$x < x_1$, $n < N+1$

$$\{H_n\} = \left[\frac{x_j (\alpha_n^4 - \beta_n^4 - 4\alpha_n^2 \beta_n^2)}{\alpha_n^2 + \beta_n^2} + \frac{\alpha_n (4\alpha_n^2 \beta_n^2 - \beta_n^4 - \alpha_n^4)}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{-4x_j \alpha_n \beta_n (\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} + \frac{\beta_n (5\alpha_n^4 - \beta_n^4 - 4\alpha_n^2 \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^i\}$$

$$\{I_n\} = \left[\frac{-4x_j \alpha_n^3 \beta_n}{\alpha_n^2 + \beta_n^2} + \frac{\beta_n (3\alpha_n^4 - \beta_n^4 - 4\alpha_n^2 \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{x_j (6\alpha_n^2 \beta_n^2 - \alpha_n^4 - \beta_n^4)}{\alpha_n^2 + \beta_n^2} + \frac{\alpha_n (\alpha_n^4 - \beta_n^4 - 8\alpha_n^2 \beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2} \right] \{d_n^i\}$$

A11

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \left[\frac{4x_j\alpha_1\beta_1(\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} - \frac{\beta_n(5\alpha_1^4 - \beta_1^4 - 4\alpha_1^2\beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2}\right]\{d_n^r\}$$

$$+ \left[\frac{x_j(\alpha_1^4 - \beta_1^4 - 4\alpha_1^2\beta_1^2)}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1(4\alpha_1^2\beta_1^2 - \beta_1^4 - \alpha_1^4)}{(\alpha_1^2 + \beta_1^2)^2}\right]\{d_n^r\}$$

$$\{I_n\} = \left[\frac{x_j(\alpha_1^4 + \beta_1^4 - 6\alpha_1^2\beta_1^2)}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1(8\alpha_1^2\beta_1^2 - \alpha_1^4 + \beta_1^4)}{(\alpha_1^2 + \beta_1^2)^2}\right]\{d_n^r\}$$

$$+ \left[\frac{-4x_j\alpha_1^3\beta_1}{\alpha_1^2 + \beta_1^2} + \frac{\beta_1(3\alpha_1^4 - \beta_1^4 - 4\alpha_1^2\beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2}\right]\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$x_1 \leq x \leq x_2$, $n < N+1$

$$\{H_n\} = (-1)^j[-x_j(\alpha_n^2 - \beta_n^2) + \alpha_n]\{d_n^r\} + [\beta_n - 2x_j\alpha_n\beta_n]\{d_n^i\}$$

$$\{I_n\} = (-1)^j[\beta_n - 2x_j\alpha_n\beta_n]\{d_n^r\} + [x_j(\alpha_n^2 - \beta_n^2) - \alpha_n]\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

A12

$n = N+1$

$$\{H_n\} = [2x_j\alpha_1\beta_1 - \beta_1]\{d_n^r\} + (-1)^j[\alpha_1 - x_j(\alpha_1^2 - \beta_1^2)]\{d_n^i\}$$

$$\{I_n\} = [\alpha_1 - x_j(\alpha_1^2 - \beta_1^2)]\{d_n^r\} + (-1)^j[\beta_1 - 2x_j\alpha_1\beta_1]\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$x > x_2 \, , \ n < N+1$

$$\{H_n\} = \left[\frac{x_j(\alpha_n^4 - \beta_n^4 - 4\alpha_n^2\beta_n^2)}{\alpha_n^2 + \beta_n^2} + \frac{(4\alpha_n^2\beta_n^2 - \beta_n^4 - \alpha_n^4)}{(\alpha_n^2 + \beta_n^2)^2}\right]\{d_n^r\}$$

$$+ \left[\frac{4x_j\alpha_n\beta_n(\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} - \frac{\beta_n(5\alpha_n^4 - \beta_n^4 - 4\alpha_n^2\beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2}\right]\{d_n^i\}$$

$$\{I_n\} = \left[\frac{-4x_j\alpha_n^3\beta_n}{\alpha_n^2 + \beta_n^2} + \frac{n(3\alpha_n^4 - \beta_n^4 - 4\alpha_n^2\beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2}\right]\{d_n^r\}$$

$$+ \left[\frac{x_j(-6\alpha_n^2\beta_n^2 + \alpha_n^4 + \beta_n^4)}{\alpha_n^2 + \beta_n^2} - \frac{\alpha_n(\alpha_n^4 - \beta_n^4 - 8\alpha_n^2\beta_n^2)}{(\alpha_n^2 + \beta_n^2)^2}\right]\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+$

$$\{H_n\} = \left[\frac{-4x_j \alpha_1 \beta_1 (\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} - \frac{n(5\alpha_1^4 - \beta_1^4 - 4\alpha_1^2\beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{x_j(\alpha_1^4 - \beta_1^4 - 4\alpha_1^2\beta_1^2)}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1(4\alpha_1^2\beta_1^2 - \beta_1^4 - \alpha_1^4)}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^i\}$$

$$\{I_n\} = \left[\frac{x_j(6\alpha_1^2\beta_1^2 - \alpha_1^2 - \alpha_1^4\beta_1^4)}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1(\alpha_1^4 - \beta_1^4 - 8\alpha_1^2\beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^r\}$$

$$+ \left[\frac{-4x_j \alpha_1^3 \beta_1}{\alpha_1^2 + \beta_1^2} + \frac{\beta_1(3\alpha_1^4 - \beta_1^4 - 4\alpha_1^2\beta_1^2)}{(\alpha_1^2 + \beta_1^2)^2} \right] \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$\underline{e}. \quad [D_b]$

$x < x_1 \, , \; n < N+1$

$$\{H_n\} = -\frac{\alpha_n}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = -\frac{\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{\alpha_n}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = -\frac{\beta_1}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \frac{-\alpha_1}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$$\{I_n\} = -\frac{\alpha_1}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \frac{-\beta_1}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

A14

$x_1 \leq x \leq x_2$, $n < N+1$

$$\{H_n\} = (-1)^j \frac{\alpha_n}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = (-1)^j \frac{\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \frac{2\beta_n}{(\alpha_n^2 + \beta_n^2)^2} \{d_n^i\}$$

$n = N+1$

$$\{H_n\} = \frac{-\beta_1}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + (-1)^j \frac{-\alpha_1}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$$\{I_n\} = -\frac{\alpha_1}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + (-1)^j \frac{\beta_1}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$$\{J_n\} = \frac{-2\beta_n}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$x > x_2$, $n < N+1$

$$\{H_n\} = -\frac{\alpha_n}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{-\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = -\frac{\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{-\alpha_n}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = -\frac{\beta_1}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \frac{-\alpha_1}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$$\{I_n\} = -\frac{\alpha_1}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \frac{-\beta_1}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

A15

$$\underline{f}. \quad [D_b']$$

$x < x_1$, $n < N+1$

$$\{H_n\} = \{d_n^r\} + \frac{-2\alpha_n\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = \frac{-(\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \frac{2\alpha_1\beta_1}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \{d_n^i\}$$

$$\{I_n\} = \frac{\alpha_1^2 - \beta_1^2}{\alpha_1^2 + \beta_1^2} \{d_n^r\}$$

$$\{J_n\} = \{0\}$$

$x_1 \leq x \leq x_2$, $n < N+1$

$$\{H_n\} = (-1)^j\{d_n^r\}$$

$$\{I_n\} = -\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = (-1)^j\{d_n^i\}$$

$$\{I_n\} = \{d_n^r\}$$

$$\{J_n\} = \{0\}$$

A16

$x > x_2$, $n < N+1$

$$\{H_n\} = \{d_n^r\} + \frac{2\alpha_n\beta_n}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = \frac{\alpha_n^2 - \beta_n^2}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \frac{-2\alpha_1\beta_1}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \{d_n^i\}$$

$$\{I_n\} = \frac{-(\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} \{d_n^r\}$$

$$\{J_n\} = \{0\}$$

g. $[D_b'']$

$x < x_1$, $n < N+1$

$$\{H_n\} = -\alpha_n\{d_n^r\} + \frac{\beta_n(3\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = \frac{\alpha_n^2(\alpha_n + \beta_n) + \beta_n^2(\beta_n - \alpha_n)}{(\alpha_n^2 + \beta_n^2)} \{d_n^r\} - \frac{2\alpha_n\beta_n^2}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

A17

$n = N+1$

$$\{H_n\} = \frac{\beta_1(\beta_1^2 - 3\alpha_1^2)}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + [-\alpha_1] \{d_n^i\}$$

$$\{I_n\} = \frac{2\alpha_1\beta_1^2}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \frac{\alpha_1^2(\alpha_1 + \beta_1) + \beta_1^2(\beta_1 - \alpha_1)}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$x_1 \leq x \leq x_2$, $n < N+1$

$$\{H_n\} = (-1)^j[-\alpha_n] \{d_n^r\} + (-\beta_n) \{d_n^r\}$$

$$\{I_n\} = (-1)^j[-\beta_n] \{d_n^r\} + \alpha_n\{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$$\{H_n\} = \beta_1\{d_n^r\} + (-1)^j[-\alpha_1] \{d_n^i\}$$

$n = N+1$

$$\{I_n\} = -\alpha_1\{d_n^r\} + (-1)^j[-\beta_1] \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

A18

$x > x_2$, $n < N+1$

$$\{H_n\} = -\alpha_n\{d_n^r\} + \frac{\beta_n(\beta_n^2 - 3\alpha_n^2)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = \frac{\alpha_n^2(\alpha_n + \beta_n) + \beta_n^2(\beta_n - \alpha_n)}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{2\alpha_n\beta_n^2}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \frac{\beta_1(3\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + [-\alpha_1] \{d_n^i\}$$

$$\{I_n\} = \frac{2\alpha_1\beta_1^2}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \frac{\alpha_1^2(\alpha_1 + \beta_1) + \beta_1^2(\beta_1 - \alpha_1)}{\alpha_1^2 + \beta_1^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

    <u>h</u>. $[D_b''']$

$x < x_1$, $n < N+1$

$$\{H_n\} = \frac{(\alpha_n - \beta_n)(\alpha_n^3 + \beta_n^3)}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{-3\alpha_n\beta_n(\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = \frac{\alpha_n(\alpha_n\beta_n^2 - \alpha_n^3 - 2\alpha_n^2\beta_n - 2\beta_n^3)}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{\beta_n^2(5\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \frac{3\alpha_1\beta_1(\alpha_1^2 - \beta_1^2)}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \frac{(\alpha_1 - \beta_1)(\alpha_1^3 + \beta_1^3)}{\alpha_1^2 + \beta_1^2} \{d_n^1\}$$

$$\{I_n\} = \frac{\beta_1^2(\beta_1^2 - 5\alpha_1^2)}{\alpha_1^2 + \beta_1^2} \{d_n^r\} + \frac{\alpha_1(\alpha_1\beta_1 - \alpha_1^3 - 2\alpha_1\beta_1 - 2\beta_1^3)}{\alpha_1^2 + \beta_1^2} \{d_n^1\}$$

$$\{J_n\} = \{0\}$$

$x_1 \leq x \leq x_2$, $n < N+1$

$$\{H_n\} = (-1)^J[\alpha_n^2 - \beta_n^2] \{d_n^r\} + 2\alpha_n\beta_n \{d_n^2\}$$

$$\{I_n\} = (-1)^J[2\alpha_n\beta_n] \{d_n^r\} + [-(\alpha_n^2 - \beta_n^2)] \{d_n^1\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = -2\alpha_1\beta_1 \{d_n^r\} + (-1)^J[\alpha_1^2 - \beta_1^2] \{d_n^1\}$$

$$\{I_n\} = (\alpha_1^2 - \beta_1^2) \{d_n^r\} + (-1)^J[2\alpha_1\beta_1] \{d_n^1\}$$

$$\{J_n\} = \{0\}$$

A20

$x < x_2$, $n < N+1$

$$\{H_n\} = \frac{(\alpha_n - \beta_n)(\alpha_n^3 + \beta_n^3)}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{3\alpha_n\beta_n(\alpha_n^2 - \beta_n^2)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = \frac{\alpha_n(\alpha_n\beta_n^2 - \alpha_n^3 - 2\alpha_n^2\beta_n - 2\beta_n^3)}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{\beta_n^2(\beta_n^2 - 5\alpha_n^2)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

$n = N+1$

$$\{H_n\} = \frac{-3\alpha_1\beta_1(\alpha_1^2 - \beta_1^2)}{\alpha_n^2 + \beta_n^2} \{d_n^r\} \frac{(\alpha_1 - \beta_1)(\alpha_1^3 + \beta_1^3)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{I_n\} = \frac{\beta_1^2(5\alpha_1^2 - \beta_1^2)}{\alpha_n^2 + \beta_n^2} \{d_n^r\} + \frac{\alpha_1(\alpha_1\beta_1^2 - \alpha_1^3 - 2\alpha_1^2\beta_1 - 2\beta_1^3)}{\alpha_n^2 + \beta_n^2} \{d_n^i\}$$

$$\{J_n\} = \{0\}$$

A21

APPENDIX B: COMPUTER PROGRAM "2PARM"

Outline of Program

1. The computer program "2PARM" is used for the elastic response analysis of a beam lying over a ground surface. The formulations in the program are obtained by idealizing the soil medium as a new two-parameter model explained in the text. The program can handle the following conditions:

<u>a</u>. Elastic soil defined by Young's modulus and Poisson's ratio.

<u>b</u>. With and without elastic uniform beam lying over a horizontal ground surface.

<u>c</u>. Horizontally layered soil medium, including a homogeneous medium, underlain by a rigid base.

<u>d</u>. A number of vertically distributed and/or simultaneously applied line loads at any location on the beam.

<u>e</u>. Plane strain conditions.

2. In the analysis the soil is divided into a number of horizontal layers with soil properties linearly varying within a layer, and the distributed load is divided into a number of trapezoidal loads.

3. The program computes the following at any location along the ground surface, beam, and interface between the layers:

<u>a</u>. Shear and moment forces induced in the beam.

<u>b</u>. Vertical and rotational displacements and forces induced in the beams.

<u>c</u>. Vertical displacements at the interfaces between the soil layers.

<u>d</u>. Contact soil reaction against the beam.

4. The computation process is explained in Figure B1. The following are the descriptions of the subroutines in the program:

INPUT   This subroutine reads in all necessary data that is related to the problem.

SKMAT   It forms the generalized stiffness matrix, $[k]$, in the equation
$[EI]\{W^{IV}\} - [N]\{W''\} + [K]\{W\} = \{P\}$.

DMAT   It forms the generalized shear matrix, $[N]$, in the equation above.

EIGEN   It computes the eigenvalues/eigenvectors for soil regions where beam is not present by using Jacobi iteration method. $Ax = x$.

B1
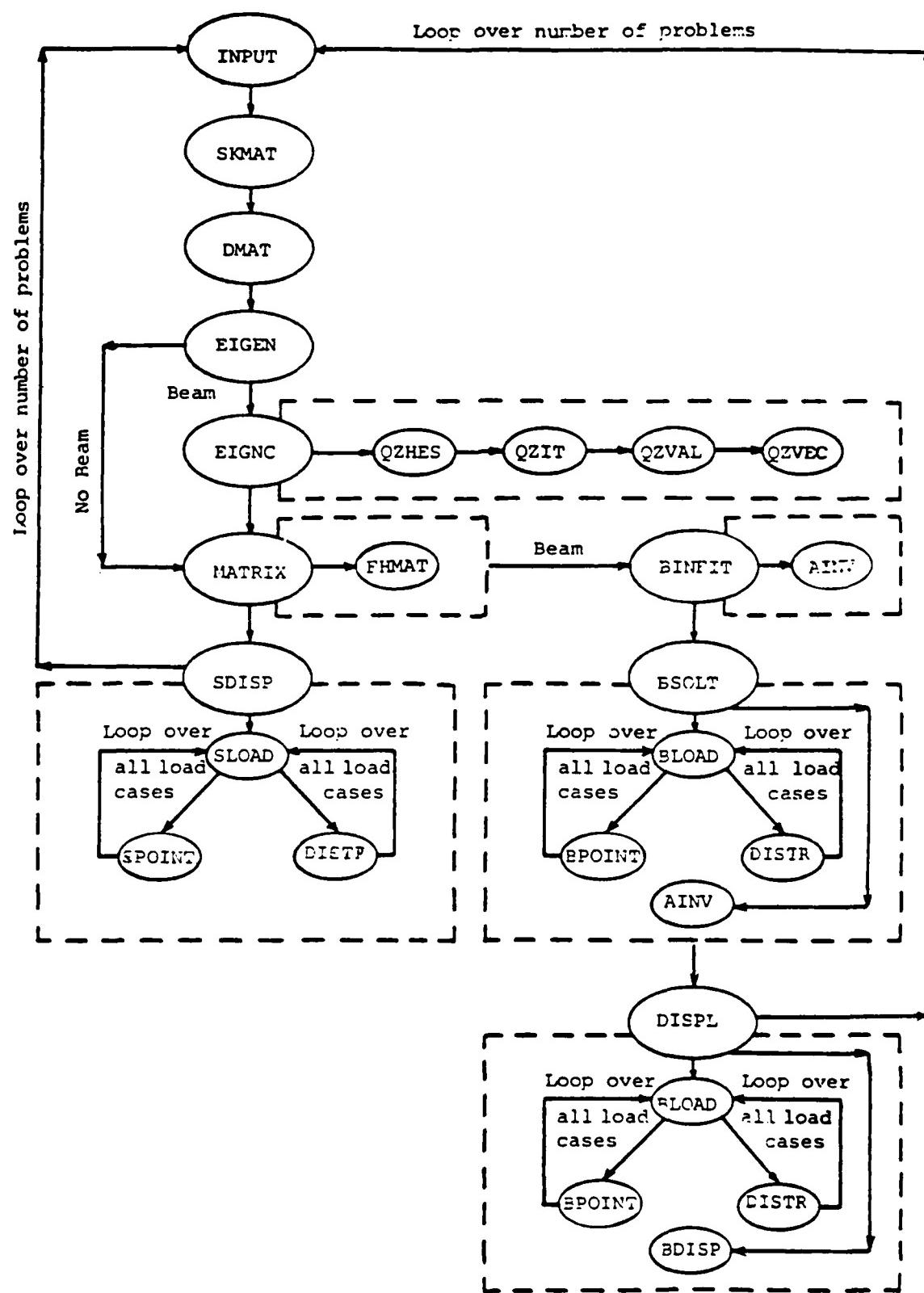
Figure B1. Flow diagram for computer program

EIGNC   It computes the eigenvalues/eigenvectors for region where beam is
          present. $Ax = Bx$.

OZHES   It reduces the generalized real eigen problem $Ax = Bx$ to an equiv-
          alent problem with A in upper Hessenberg form and B in upper
          triangular form, using orthogonal transformations.

QZIT   It reduces A and B as computed in subroutine OZHES to equivalent
          problem with A now in quasi-upper triangular form and B still in
          upper triangular form.

OZVAL   A and B from subroutine QZIT is used to compute quantities whose
          ratios give the generalized eigenvalues.

OZVEC   It determines the eigenvectors corresponding to the set of specified
          eigenvalues as found in subroutine OZVAL.

MATRIX   Formation of matrices using the eigenvalues/eigenvectors as found in
          subroutine EIGEN and EIGNC.

FHMAT   Formation of matrices in the solution expression.

SDISP   Computes displacements for cases without beam.

SLOAD   This subroutine is called by subroutine SDISP to compute the solu-
          tion due to all load cases.

SPOINT   Computes solution due to point loads on soil surface.

DISTR   Computes the solution due to distributed loads.

BINFIT   Computes the solution for point load on infinite beam.

BSOLT   Computes the unknown constants in solution expressions for the case
          with beam.

BLOAD   Computes the particular solution due to all load cases on beam.

BPOINT   Computes the particular solution for point loads on infinite beam.

AINV   Subroutine for real matrix inversion.

DISPL   Computes the solution for soil regions outside the loaded beam.

BDISP   Computes the displacement, moment, shear, and contact forces for regions occupied with beam.

Inputs

5. The input variable names are explained in Figure B2. The following is the input card format for Program 2PARM:

Card Group 1   Identification Card - FORMAT (20A4)

1 - 80   Alphanumeric identifying information to be printed as titles on the output

Card Group 2   Control cards - FORMAT (I10)

1 - 10   N - Number of soil layers (maximum = 10)

Card Group 3   Material property information

First card: Soil material property information - FORMAT (F10.3)

1 - 10   Poisson's ratio of soil

Second card: Young's modulus of soil assumed linearly varying with depth - FORMAT (2F10.3)

1 - 10   E1(I) - Young's modulus at top fibre of soil layer I (psf)

11 - 21   E2(I) - Young's modulus at bottom fibre of soil layer I (psf)

Additional card must be supplied for each soil layer.

Third card: Soil layer thickness - FORMAT (7F10.3)

1 - 10   THK(I) - Soil thickness of the Ith layer (ft)

Repeat for the remaining six fields and continue on additional cards if necessary.

Fourth card: Beam geometry and material property - FORMAT (3F10.3)

1 - 10   BL - Length of beam (ft)
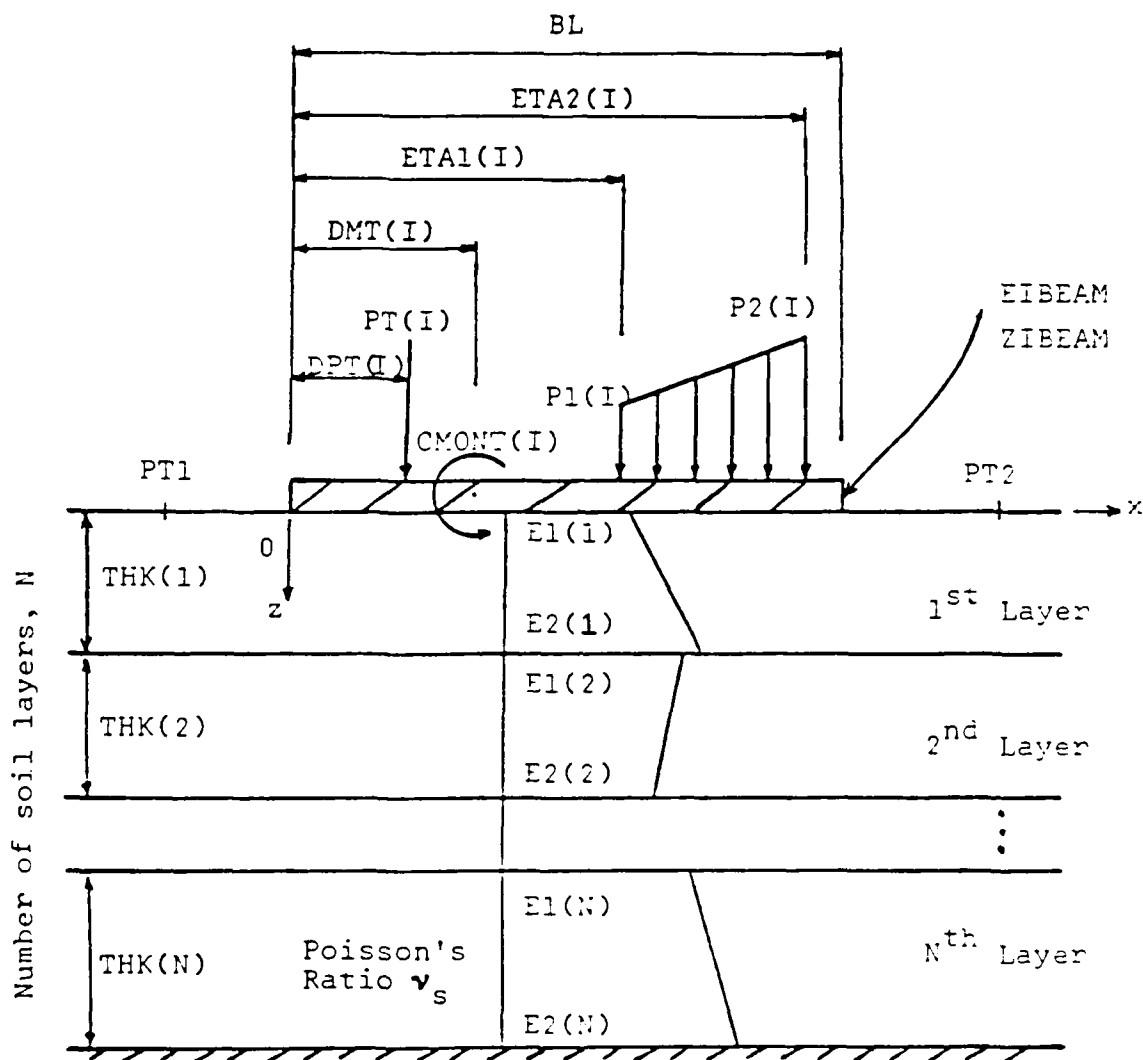
Figure B2. Input data notations used in computer program

| 11 - 20 | EIBEAM – Young's modulus of beam (psf) |
| 21 - 31 | ZIBEAM – Moment of inertia of beam (ft ** 4) |

<u>Card Group 4</u> Loading conditions

First card: Total number of loads – FORMAT (3I10)

1 - 10	NPT – Number of concentrated point loads
11 - 20	NMONT – Number of applied moments
21 - 30	NDISTR – Number of distributed loads

Second card: Point load magnitude (downward positive) and their applied
locations – FORMAT (2F10.3)

| 1 - 10 | PT(I) – Magnitude of point load I (lb) |
| 11 - 20 | DPT(I) – Location of applied load I (ft) |

Additional card must be supplied for each point load.

Third card: Moment magnitude (counter clockwise is positive) and their applied locations – FORMAT (2F10.3)

1 – 10   CMONT(I) – Magnitude of applied moment I (ft-lb)

11 – 20   DMT(I) – Location of applied moment I (ft)

Additional card must be supplied for each applied moment.

Fourth card: Distributed loads (assumed linear variation; downward is positive) and their applied locations – FORMAT (4F10.3)

1 – 10   P1(I) – Magnitude at near end of distributed load I (lb)

11 – 20   P2(I) – Magnitude at far end of distributed load I (lb)

21 – 30   ETA1(I) – Near end location of distributed load I (ft)

31 – 40   ETA2(I) – Far end location of distributed load I (ft)

Additional card must be supplied for each distributed load.

Note: Fields E and F are interchangeable. If a number is more suited to one field, that field may be used.

Card Group 5 Locations where solution is desired – FORMAT (3F10.3)

1 – 10   PT1 – Near end point where solution is to be computed (ft)

11 – 20   PT2 – Far end point where solution is to be computed (ft)

21 – 30   DELTA – Regular interval between PT1 and PT2; where the solution is to be computed (ft)

Card Group 6 Modification factors – FORMAT (2F10.3)

1 – 10   FE – Modification factor for axial forces in soil column

11 – 20   FG – Modification factor for shear forces in soil column

If this card is blank, the modification factors are computed from Equation 49b.

6. Several problems may be analyzed in one run. The data set for each problem commences with an identification card. Two consecutive blank cards are required to stop program.

Outputs

7. The program prints the following outputs:

a. Input and generated data.

b. Displacement (ft), shear (k), moment (k-ft) and contact pressure (ksf) at each location as specified, card group 5.

B6

8. The case considered for an example run is illustrated in Figure B3.
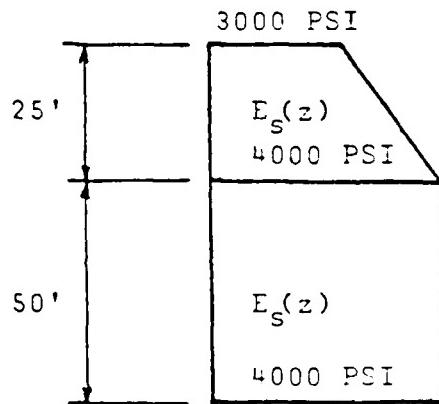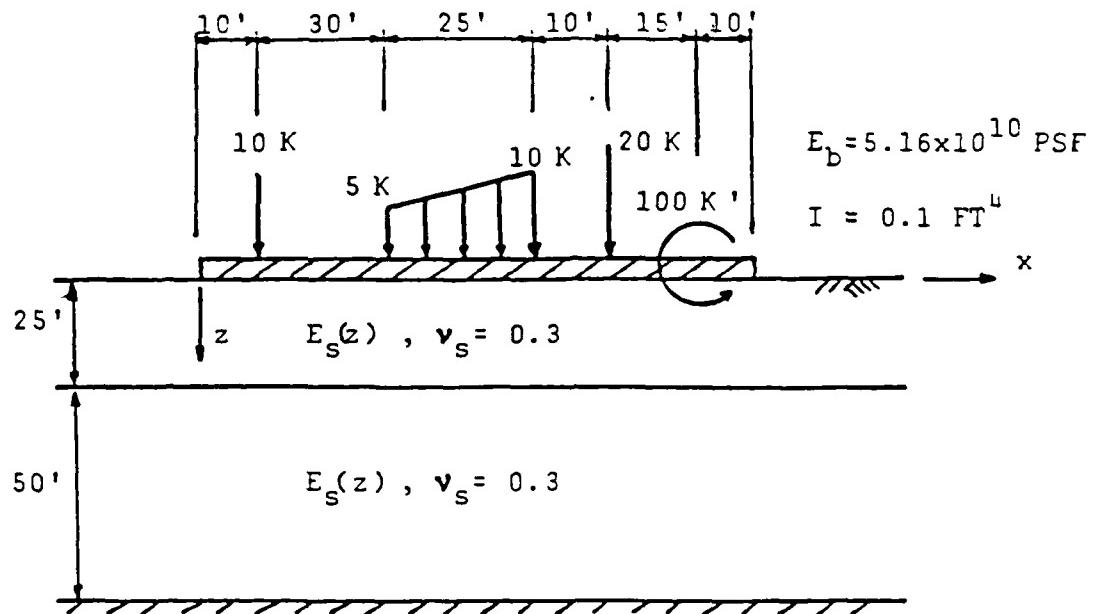
Figure B3. Case considered in example run

9. The input data setup and computer outputs produced by Program 2PARM for a beam with multiple loading conditions follow:

EXAMPLE RUN

TEST RUN 1. -- BEAM WITH MULTIPLE LOADING CONDITIONS
```
            2
          0.3
         25.0      4.32E5    5.76E5
         50.0      5.76E5    5.76E5
        100.0      5.16E10    0.1
            2         1         1
      10000.0      10.0
      20000.0      75.0
        1.0E5      90.0
       5000.0      1.0E4     40.0
          0.0     100.0      50.0
          1.0       1.0
END OF TEST RUN
            0
```

I N P U T   I N F O R M A T I O N

TEST RUN 1 -- BEAM WITH MULTIPLE LOADING CONDITIONS

NUMBER OF SOIL LAYERS, N	=	2
POISSON'S RATIO OF SOIL	=	0.300
LENGTH OF BEAM, BL	=	100.000 FEET
YOUNG'S MODULUS OF BEAM	=	0.516D+11 PSF
MOMENT OF INERTIA OF BEAM	=	0.100 FT.**4
NEAR END PT WHERE SOLUTION IS DESIRED	=	0.000 FEET
FAR END PT WHERE SOLUTION IS DESIRED	=	100.000 FEET
INTERVALS BETWEEN POINTS	=	50.000 FEET
SOIL COLUMN AXIAL FORCES MOD. FACTOR	=	1.000
SOIL COLUMN SHEAR FORCES MOD. FACTOR	=	1.000

SOIL LAYER #	THICKNESS	E1(I)	E2(I)
1	25.0000	0.4320D+06	0.5760D+06
2	50.0000	0.5760D+06	0.5760D+06

```
AT DISTANCE X =          0.00  FEET
-------------------------------------

SOIL LAYER #             DISPLACEMENT
                             (FEET)

        1               0.721710D-01
        2               0.535073D-01


CONTACT PRESSURE =       0.458358D+00   KSF
ROTATION         =      -0.412303D-02   RADIANS
MOMENT           =      -0.699310D-13   KIP-FEET
SHEAR FORCE      =      -0.248574D+00   KIPS




AT DISTANCE X =          50.00  FEET
-------------------------------------

SOIL LAYER #             DISPLACEMENT
                             (FEET)

        1               0.247246D+00
        2               0.141223D+00


CONTACT PRESSURE =       0.327420D+01   KSF
ROTATION         =      -0.107709D-02   RADIANS
MOMENT           =       0.110158D+04   KIP-FEET
SHEAR FORCE      =       0.257225D+02   KIPS




AT DISTANCE X =          100.00  FEET
-------------------------------------

SOIL LAYER #             DISPLACEMENT
                             (FEET)

        1               0.981417D-01
        2               0.671236D-01


CONTACT PRESSURE =       0.812124D+00   KSF
ROTATION         =       0.468257D-02   RADIANS
MOMENT           =      -0.524483D-13   KIP-FEET
SHEAR FORCE      =      -0.178934D+01   KIPS
```

```
NUMBER OF CONCENTRATED POINT LOADS =    2


     POINT LOAD ( 1) =            0.1000D+05  LBS.  AT X =        10.000   FEET.
     POINT LOAD ( 2) =            0.2000D+05  LBS.  AT X =        75.000   FEET.



NUMBER OF APPLIED MOMENTS =    1


   MOMENT ( 1) =            0.1000D+06  FT-LBS  AT X =        90.000   FEET.



NUMBER OF DISTRIBUTED LOADS =    1


   DISTRIBUTED LOAD ( 1) :

   P1   =         0.5000D+04  LBS.        P2   =         0.1000D+05  LBS.
   ETA1 =            40.000   FEET        ETA2 =            65.000   FEET
```

END

DTIC

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